Name:

Enrolment No:

UPES

End Semester Examination, May 2024

Course: Mathematics II **Semester:** II

Program: B. Tech BT/FT/BME **Duration:** 3 Hours

Course Code: MATH1061 **Max. Marks:** 100

Instructions: Read the following instructions carefully:

1. Mention name and enrolment number at the top of question paper.
2. Attempt all questions from section A.
3. Attempt any four questions from section B.
4. In section C, Q7 has an internal choice.
5. In section D, Q9 has an internal choice.

S. N.	Section A Short answer questions/ MCQ/T&F (20Qx1.5M= 30 Marks)	Marks	COs
1.	The value of $\int_0^1 \int_0^x (x^2+y^2)\,dydx$ is: a. $1/2$ b. $1/3$ c. $-1/2$ d. $-1/3$	1.5	CO1
2.	If double integration in cartesian coordinates is given by $\iint f(x,y)dxdy$ then the value of the same integral in polar form is: a. $\iint f(r\cos\theta, r\sin\theta)\, r^2 dr d\theta$ b. $\iint f(r\sin\theta, r\cos\theta)\, r^2 dr d\theta$ c. $\iint f(r\cos\theta, r\sin\theta)\, r dr d\theta$ d. $\iint f(r\sin\theta, r\cos\theta)\, r dr d\theta$	1.5	CO1
3.	The divergence of a vector $xi+yj+zk$ is: a. 4 b. 0 c. 1 d. 3	1.5	CO1
4.	A vector point function $\bar{F}$ is said to be irrotational if $Div\ \bar{F} = 0$. a. True b. False	1.5	CO1
5.	The value of $\int_0^1 dx \int_0^2 dy \int_1^2 x^2yz\, dz$ is:	1.5	CO1

	a. 0 b. $\frac{2}{3}$ c. 1 d. -1		
6.	If $f(x)\ and\ g(x)$ are analytic functions, then: a. $\frac{f(x)}{g(x)}$ is always analytic. b. $\frac{f(x)}{g(x)}$ is analytic, whenever $g(x) \neq 0$. c. $\frac{f(x)}{g(x)}$ is analytic, whenever $f(x) \neq 0$ d. None of the above.	**1.5**	**CO2**
7.	The sufficient condition for $f(z)$ to be analytic is : a. $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ b. $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ c. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ d. None of these	**1.5**	**CO2**
8.	Define Harmonic function and mention its one property.	**1.5**	**CO2**
9.	What do you understand by simply and multiply connection region?	**1.5**	**CO2**
10.	If $f(z) = \frac{e^{2z}}{(z+i)(z-i)}$, then the function $f(z)$ is not analytic at: a. $z = i$ b. $z = -i$ c. $z = i, z = -i$ d. Analytic everywhere	**1.5**	**CO2**
11.	If $f(z)$ is analytic inside and on a closed curve C and if a is any point inside C, then, mention the Cauchy integral formula.	**1.5**	**CO2**
12.	For the closed integral $\int_C \frac{4-3z}{z(z-1)(z-2)}\ dz$, where C is the circle $\lvert z\rvert = \frac{3}{2}$, which statement is true: a. $z = 0, z = 1, z = 2\ lie\ inside\ C.$ b. $z = 0\ only\ lie\ inside\ C.$ c. $z = 0, z = 1\ lie\ inside\ C\ \&\ z = 2\ lies\ outside\ C.$ d. None of these	**1.5**	**CO2**
13.	A differential equation is said to be ordinary if it has: a. One dependent variable. b. More than one dependent variable. c. One independent variable d. More than one independent variable.	**1.5**	**CO3**
14.	Integrating factor of the differential equation $\frac{dx}{dy} + \left(\frac{1}{ylogy}\right)x = \frac{1}{y}$ is:	**1.5**	**CO3**

	a. y b. $logy$ c. $ylogy$ d. $1/y$		
15.	The solution of $(D^2-4)y=0$ is: a. $Ae^{2x}+Be^{4x}$ b. $Ae^{2x}+Be^{-2x}$ c. $Ae^{4x}+Be^{-4x}$ d. $Acos2x+Bsin2x$	**1.5**	**CO3**
16.	Wronskian of $y_1=cos2x,\ y_2=sin2x$ is given by: a. 1 b. -1 c. 2 d. -2	**1.5**	**CO3**
17.	Differential equation of the form $a_0x^n\frac{d^ny}{dx^n}+a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+a_2x^{n-2}\frac{d^{n-2}y}{dx^{n-2}}+\ldots\ldots\ldots\ldots\ldots\ldots+a_{n-1}x\frac{dy}{dx}+a_ny=Q(x)$ is known as: a. Legendre's differential equation b. Cauchy's Linear differential equation c. Leibnitz linear differential equation d. None of these	**1.5**	**CO3**
18.	The particular integral of $(D^2+9)y=sin4x$ is: a. $\frac{1}{7}cos4x$ b. $-\frac{1}{7}cos4x$ c. $\frac{1}{7}sin4x$ d. $-\frac{1}{7}sin4x$	**1.5**	**CO3**
19.	Mention the order and the degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^2+y=0$	**1.5**	**CO3**
20.	Mention the name of the nonlinear equation $\frac{dy}{dx}+Py=Qy^n$.	**1.5**	**CO3**

Section B
(4Qx5M=20 Marks)

Attempt any 4 questions

Q 2	Find the analytic region of $f(z)=(x-y)^2+2i(x+y)$	**5**	**CO2**
Q 3	For what values of a and b the differential equation $(y+x^3)dx+(ax+by^3)dy=0$ is exact? Also, find the solution of the equation.	**5**	**CO3**
Q 4	Investigate whether the function $u(x,y)=x^3-3xy^2+3x^2-3y^2+1$ is harmonic.	**5**	**CO2**

Q 5	Find the complementary function of $(D^4 - 2D^3 + D^2)y = 0$.	**5**	**CO3**
Q 6	Evaluate $\int_{10}^{1}\int_{0}^{1/y} ye^{xy}\, dxdy$.	**5**	**CO1**
	Section C **(2Qx15M=30 Marks)**		
Q 7	Apply method of variation of parameters to solve the differential equation: $$(D^2 + 1)y = \frac{1}{sinx}$$ **OR** Find the solution of following Cauchy Euler equation, $$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$$	**15**	**CO3**
Q 8	Change the order of integration and hence evaluate $\int_{0}^{1}\int_{x^2}^{2-x} xy\ dydx$.	**15**	**CO1**
	Section D **(2Qx10M=20 Marks)**		
Q 9	Reduce the equation $\frac{dy}{dx} + \frac{2y}{x} = y^2x^2$ to the linear equation of the form $\frac{dy}{dx} + Py = Q$ where P & Q are the functions of x only or constant, hence find the solution. **OR** Find the integrating factor of the equation $\frac{dy}{dx} - \left(\frac{3}{x}\right)y = x^3$ and solve it for y, if $y(1) = 4$.	**10**	**CO3**
Q 10	Apply Cauchy integral formula to evaluate $\int_C \frac{z}{(z-1)(z-2)}\, dz$, where C is $\lvert z - 2\rvert = \frac{1}{2}$.	**10**	**CO2**