Name: Enrolment No:	

UPES

End Semester Examination, December 2023

Course: Mathematics I **Semester:** I

Program: B.Tech. BT/FT/BME **Time** : 03 hrs.

Course Code: MATH1048 **Max. Marks:** 100

Instructions: Attempt all questions. In section C there is an internal choice in Q 2. In section D there is an internal choice in Q 2.

S. No.	Section A Short answer questions/ MCQ/T&F (20Qx1.5M= 30 Marks)	Marks	COs
Q 1	$\int_2^\infty \frac{2}{X^2}\,dx$ is equal to: a. 1 b. 0 c. -1 d. 1/2	1.5	CO1
Q 2	Expansion of $sinx$ is: a. $x+\frac{x^3}{3!}+\frac{2x^5}{15}+\cdots\ldots..$ b. $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots\ldots..$ c. $x+\frac{x^3}{3!}-\frac{2x^5}{15}+\cdots\ldots..$ d. $x+\frac{x^3}{3!}-\frac{x^5}{5!}+\cdots\ldots..$	1.5	CO1
Q 3	For what value of x the function $y=x^2-4x$ has the maximum or minimum value?	1.5	CO1
Q 4	The value of $B\left(\frac{1}{3},\frac{2}{3}\right)$ is: a. $\frac{\pi}{8}$ b. $\frac{\pi}{16}$ c. $\frac{\pi}{2}$ d. $\frac{1}{\sqrt{2}}\pi$	1.5	CO1
Q 5	Define Rolle's Theorem.	1.5	CO1
Q 6	The P series $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}\ldots\ldots.$ is convergent if: a. $p\le 1$ b. $p>1$	1.5	CO2

	c. $p = 0$ d. None of these		
Q 7	Mention the statement of D'Alembert's test of convergence.	**1.5**	**CO2**
Q 8	The function $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ is: a. Even b. Odd	**1.5**	**CO2**
Q 9	Mention two examples of periodic functions with their period.	**1.5**	**CO2**
Q 10	Test the convergence of $u_n = \frac{n^2}{n!}$	**1.5**	**CO2**
Q 11	The P series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p}$ ... . is divergent if: e. $P \leq 1$ f. $P > 1$ g. $P = 0$ h. None of these	**1.5**	**CO3**
Q 12	If $f(x, y) = xy$ then df is equal to: a. $xdx + ydy$ b. $ydx + xdy$ c. $dx + dy$ d. $dx - dy$	**1.5**	**CO3**
Q 13	If $f(x, y) = x^3 + x^2yz^2 + y^3$ then $\frac{\partial f}{\partial x}$ at $(-1, -1, -1)$ is: a. 0 b. 5 c. 3 d. -1	**1.5**	**CO3**
Q 14	$\lim_{x \to 0} \frac{sinx}{x}$ is: a. 1 b. $\frac{\pi}{2}$ c. 0 Not defined	**1.5**	**CO3**
Q 15	Determine the critical points for the function $f(x) = 2 + 2x + 2y - x^2 - y^2$.	**1.5**	**CO3**
Q 16	If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ find A^2.	**1.5**	**CO4**
Q 17	The matrix $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a: a. Scalar matrix b. Diagonal matrix c. Unit matrix d. Square matrix	**1.5**	**CO4**
Q 18	The rank of the matrix $A = \begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$ is: a. 1	**1.5**	**CO4**

	b. 2 c. 3 d. 4		
Q 19	If $A^3 + 5A^2 - 6A + I = O$ then find A^{-1}.	**1.5**	**CO4**
Q 20	What do you mean by modal matrix.	**1.5**	**CO4**
	Section B **(4Qx5M=20 Marks)**		
Q 1	Apply Maclaurin series expansion to expand the function $f(x) = (1+x)^n$ in increasing power of x.	**5**	**CO1**
Q 2	Test the convergence of the series: $$\sum_{n=1}^{\infty} \frac{(4n^2 - n)}{(n^3 + 9)}$$	**5**	**CO2**
Q 3	If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$.	**5**	**CO3**
Q 4	Find the rank of the following matrix by reducing it to Echelon form. $$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$	**5**	**CO4**
	Section C **(2Qx15M=30 Marks)**		
Q 1	Obtain the half range sine series for the function $f(x) = x^2$ in the interval $0 < x < 3$.	**15**	**CO2**
Q 2	Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ and hence obtain A^{-1}. **OR** Find all the eigen values and eigen vectors of the matrix: $$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$	**15**	**CO4**
	Section D **(2Qx10M=20 Marks)**		
Q 1	Test for consistency and solve the system of equations: $$5x + 3y + 7z = 4$$ $$3x + 26y + 2z = 9$$ $$7x + 2y + 10z = 5$$	**10**	**CO4**
Q 2	Find the volume of greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **OR** Examine $f(x,y) = x^3 + y^3 - 3axy$ for maximum and minimum values.	**10**	**CO3**