MAJOR PROJECT REPORT

On

Creation Of Flight Control Data Lookup Table For A Flight Simulator Using CFD Analysis For A Fighter Aircraft

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April, 2013

CERTIFICATE

I hereby certify that the work which is presented in the project report entitled "**Creation Of Flight Control Data Lookup Table For A Flight Simulator Using CFD Analysis For A Fighter Aircraft**" in partial fulfilment of the Bachelors in Aerospace Engineering, submitted to the Department of Aerospace Engineering, University of Petroleum and Energy Studies, Dehradun is an authentic record of my own work carried out during a period from July 2012 to April 2013 under the supervision of Asst. Prof. Sourabh Bhat, Department of Aerospace Engineering.

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This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

(Asst. Prof. Sourabh Bhat)	(Dr. Om Prakash)	Reviewer
Supervisor	Head of Department	

Date: - April 18, 2013.

ACKNOWLEDGEMENTS

We extend our sincere thanks to our Project Supervisor and Activity In-charge Asst. Prof. Sourabh Bhat, Department of Aerospace Engineering for his excellent professional guidance which was a great source of strength for me to accomplish the given assignment.

We are also thankful to Asst. Prof. Karthik Sundarraj, Course Coordinator, Department of Aerospace Engineering for extending his cooperation in this regard from time to time.

ABSTRACT

A flight simulator must mimic the actual flight as closely as possible to provide the same feel of piloting the real aircraft. To achieve this, a complete flow analysis including turbulence and other complicated phenomenon of actual flight must be incorporated into the simulator. These flight conditions can be calculated by using CFD analysis. However, doing CFD analysis on a real time basis is not possible, given the amount of time and memory required for do CFD simulations. A more feasible approach is to simulate a priori, and create "lookup tables" which can be quickly referred to by the simulator software, to calculate the flight mechanics by simple Newton's laws of motion.

In this project there are three stages. The first stage is modeling a MIG variant aircraft, with exact dimensions, along with assembly of control surfaces and hinge axis of control surfaces. But before coming on to a full-scale MiG variant this complete procedure was performed on a rough and less precise model of the Boeing 747 for gaining familiarity with the complete process.

The second stage consists of CFD analysis of the aircraft with many configurations of flight control surfaces and various flight regimes.

The third stage is made up of creation of plots for the real flight control and making lookup tables for the simulator.

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NOMENCLATURE

T	Thrust
D	Drag
W	Weight of aircraft
m	Mass of aircraft
L	Lift
α_T	Inclination of thrust by flight path
θ	Inclination of flight path with horizontal axis
α	Angle of attack
V	Instantaneous value of aircraft's flight velocity
$\frac{dV}{dt}$	Acceleration along the flight path
r_c	Radius of curvature
δ_e	Elevator deflection
δ_r	Rudder deflection
δ_a	Aileron deflection
C_L	Coefficient of Lift
C_D	Coefficient of Drag
C_{Mx}	Coefficient of Moment about X-axis
C_{My}	Coefficient of Moment about Y-axis
C_{Mz}	Coefficient of Moment about Z-axis

CHAPTER 1: BACKGROUND

1.1 Simulator

A flight simulator is the equipment that synthesizes the flight conditions and environmental factors using powerful computers. These factors ranges from the flight governing equations to phrasing the actions and reactions in case of atmospheric disturbances such as the gusts and turbulences. These flight simulators are deployed for flight training of pilots operating civil as well as military planes, researching aircraft features and quality and control systems and sometimes are even used for recreational functions.

The detailing of the flight simulator depends on the need which it is to serve and accordingly it deploys the hardware which in cases can be a personal computer for commercial gaming or even a supercomputer for mocking spacecrafts, replicating its complex cockpit consoles with broad atmospheric visuals, and the whole set up hoisted on a multi-degree-of-freedom chamber responding to the trainee inputs and simulated flight conditions.

The most extensive use by far remains the pilot training for flying as well as combat readiness or disaster management in case of civil aircrafts.

1.2 Project Issue

The aircraft flying and movements are controlled through the control surfaces. Any deflection or alteration from the neutral position of the control surfaces would create some forces and moments on the aircraft that could potentially alter a powered straight-flight-path of an aircraft. These forces and moments with their potentials are calculated and learned for effective deployment in aircraft flight controls.

This project is focused on this very purpose of study of the effects of different levels of deflection of various control surfaces and their corresponding effects on aircraft, so the pilots can get familiar with the action-reaction balance of stick movements and the aircraft manoeuvres.

This is done by calculating one by one the effects of every control surface at various degrees of deflection. The moments are noted for a set of fixed interval deflections of the control surfaces. Then the values are used to generate a transform function for the relationship between the degree of deflection and corresponding moments generated. Same when done for every control surface would give a transform function of every control surface which would then be compiled to give look up tables that would define reactions and aircraft movements for their corresponding pilot inputs and atmospheric disturbances.

CHAPTER 2: MODELLING

Computational fluid dynamics (CFD) is the software for mathematical solution deriving and analysis of fluid flow problems that includes flow over surfaces of objects and requires in-depth command of fluid mechanics. Only powerful andlarge processors and super-computers can process the solutions to such fluid dynamics problems that goes up to the spectrum of transonic, non-laminar and supersonic flows.

It involves generation of system of "ordinary differential equations" for addressing the different spectrums of mathematical problems through discretization done in the space, while for the non linear algebraic equations implicit approach is adopted to integrate the other differential equations. Then using iterations a system of linear equations are generated which could be non-symmetric or indefinite depending upon the flow. Solutions to these set of equations are generally too large and thus the iterative methods are put into action examples of which are the Krylov Subspace method that uses precondition to reduce the residual over successive subspaces encountered during the preconditioning, or the option such as the successive over-relaxation method.There are also multigrid options that unlike the older solvers can effectively bring down not only the high frequency residual but also the low frequency residuals by similar factors across all components which means the iterations are even less bounded by the meshing processes.

2.1 SolidWorks

SolidWorks is 3D CAD software which allow to quickly design 3D models. It has user interface and powerful design capabilities which provide accuracy. It's easy to learn and use as is user friendly. Its built-in intelligence accelerates the design process. For modelling using SolidWorks a 2D model is drafted first with fundamental figures such as lines and arcs after units and dimensions are induced. In SolidWorks the interrelations dictates the geometry. Further, any assemblies conducted in solidworks are also driven by internal conditions such as perpendicularity and tangency. [4]

Various features used in designing:

- Mirror
  'Mirror' is the function which is used when a mirror image of a component designed has to be placed by its side i.e. when symmetrical section has to be designed.
- Extrude
  'Extrude' is used when a model or its component is designed in a 2 dimensional coordinates and it has to be transformed into a 3-D coordinate.
- Fillet
  'Fillet' is of fore most importance as it makes sure that the edges of the components are not sharp but smooth and curved.
- Loft
  'Loft' feature allows generation of complex geometry in a single feature. It does this by interpolating surfaces between various cross-sections of a model. These cross-sections can be sketches, faces, or edges. [5]

2.2 Gambit

Different CFD problems require different mesh types, and GAMBIT provides all the options we needed to explore under one interface. GAMBIT is solely used for meshing and it creates simple geometry than divides in parts. It has tools to address geometry, mesh and zoning. Geometry lets us create lines and faces with volumes and groupings. Meshing further allows to mesh faces or edges or entire volumes. Zones lets us define the boundary conditions in this pre-processor.

Tools that came in handy while using GAMBIT were "Mesh volumes" for creating mesh nodes throughout the volume, "Smooth volume meshes" for adjust volume mesh node positions for uniformity of spacing the nodes, "Set volume element type" that specifies the type of volume element used in the model and "Link Unlink volume meshes" for creating as well as removing mesh hard links in the volumes. [6]

2.3 Fluent

Fluent provides the tools to model the broad physical properties such as the turbulence and heat transfer as well as the complex industrial processes like the airflow over wings and surface of vehicles and aircrafts and the combustion in a furnace.

The commands in Fluent that cane in handy during the phase of formation of geometry were "Form volume(command)" that creates a volume from existing faces as well as edges. "Create volume(command)" that creates a volume in one of the several primitive shapes. "Boolean Operations(command)" that combines, intersects or subtracts volumes and lastly the "Blend volume", this command rounds or trims the volume edges. [7]

2.4 Control surfaces

2.4.1 AILERON:

These are the control surfaces attached to the trailing edge of the wing of a fixed-wing aircraft and are used to control the aircraft in roll. Both the ailerons are interconnected so that when one goes up, the other goes down. The down going aileron increases the lift whereas in the up going aileron decreases it which produce a rolling moment about the aircraft longitudinal axis and makes the aircraft laterally stable. Use of aileron for roll moment also produce yaw moment due the change in drag on both the wings (known as adverse yaw).

2.4.2 RUDDER:

This control surface is similar to the aileron and elevator. It is mainly used to counter the adverse yaw due to aileron and hence help in yaw motion of aircraft about the vertical axis. It is a flat plate attached to the tail of the aircraft.

2.4.3 ELEVATORS:

This control surface is at the rear of the aircraft and controls the pitch moment about the lateral axis and keeps the aircraft stable about the longitudinal axis. As they change the pitch they also change the angle of attack and hence the lift. The rear wing (i.e. stabilizer) to which elevators are attached have the opposite effect to that of the wing (downward lift).

2.4.4 FLAPS:

These are hinged surfaces on the trailing edge of the wing and reduce the stalling speed of the aircraft when extended so as to make sure the safe flight at lower speeds. Extending the flap increases the camber of the wing aerofoil and thus increases the lift coefficient which increases the lift at slower speed. Extending also increase the drag which may be beneficial during the approach and landing as it helps to slow down the aircraft. [3]

2.5 Equations of motion

Following equations governs the aircraft's translation motion through air, and thus must first be established. Solving for a curvilinear flight path gives equations of motion of an airplane in translational flight. [1]

$$T\cos\alpha_{\tau} - D - W\sin\theta = m\frac{dV}{dt}$$

$$L + T\sin\alpha_{\tau} - W\cos\theta = m\frac{V^2}{r_c}$$

2.6 Moments

After lift and drag, surface pressure and shear stress distributions create moments. These moments are effective on wings and aft section only. [2]

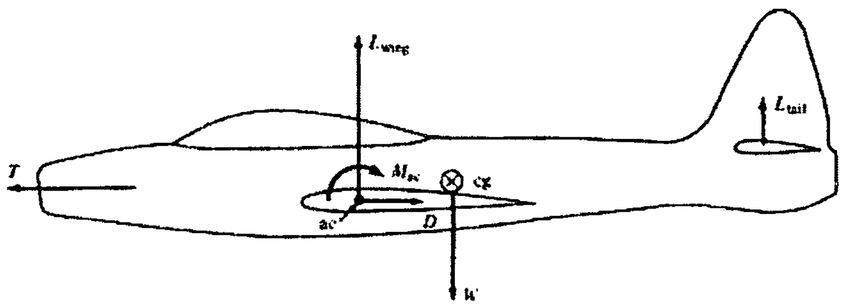

*Figure 2.1: **Contribution of moments along centre of gravity of airplane***

CHAPTER 3: PRELIMINARY STUDY

3.1 Cad Model

The designing of the CAD model is done in the SolidWorks. The designing was executed using the loft feature of SolidWorks. Different ellipses were drawn around an axis that served as the various cross-sections of a Boeing 747, and were then formed into a model using loft feature. Then nose shaped and smoothened using the constraints within the loft feature only and same goes with the tail. Then, using the spline feature, the aerofoil was imported and sized in line with the fuselage we created by lofting. The aerofoil chord at the wing tip was then reduced proportionally and lofted with the root-aerofoil, and then using "mirror image" feature the set of the aircraft wings was designed. Similarly the elevators were also designed. For the rudder, an aerofoil was drawn using spline on fuselage topplane,another resized aerofoil was drawn at some distance and were both lofted to create our rudder. To finish it off we used "fillets" for smoothening the surfaces. The aircraft model side view with dimensions for full aircraft length and tail height. aircraft top view showing the wing sizes.

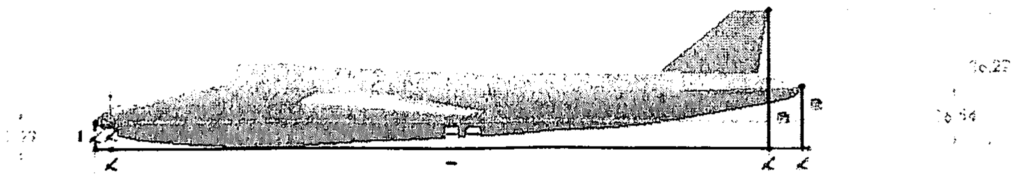

Figure 3.1: Side View

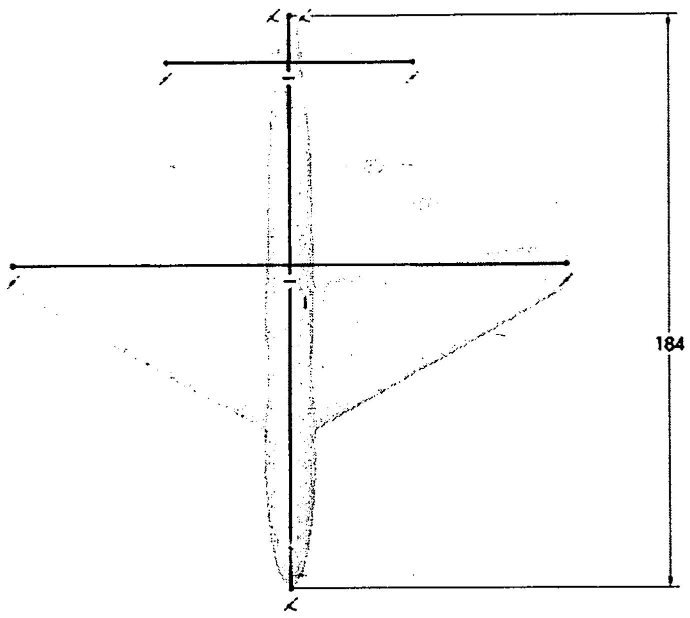

Figure 3.2: Top VIEW

3.2 Meshing

Meshing of the complete was done in GAMBIT software. The CAD model in IGES format from SolidWorks was imported in the gambit. A spherical volume was created with its centre considering with the centre of aircraft and had a radius 25 times the length of the aircraft. From this volume, the real volume or the aircraft volume was subtracted to acquire a single volume. This final volume was face meshed with Tri pave meshing giving different spacing to different faces in order to develop a mesh without any screwed or inverted elements. After the surface mesh was completed without any of those elements, volume was meshed with Tet/HybTGrid meshing. The complete was meshed and re-meshed until there were no errors encountered. After completing the meshing process the boundary conditions were fed i.e. aircraft was taken as the wall and the sphere as the pressure farfield. Then this file was exported to fluent.

3.3 Solver

FLUENT software was employed for the flow simulation. The first step was to read the case and check for problems in the mesh. From our little experience coupled solver was used as it gave better results over the Mach 0.85 and corresponding compressible flow. Explicit formulation takes up considerably less memory but involves more time for solving. With invarying boundary conditions and a constant mach number throughout the flow we had in our hands a steady problem.

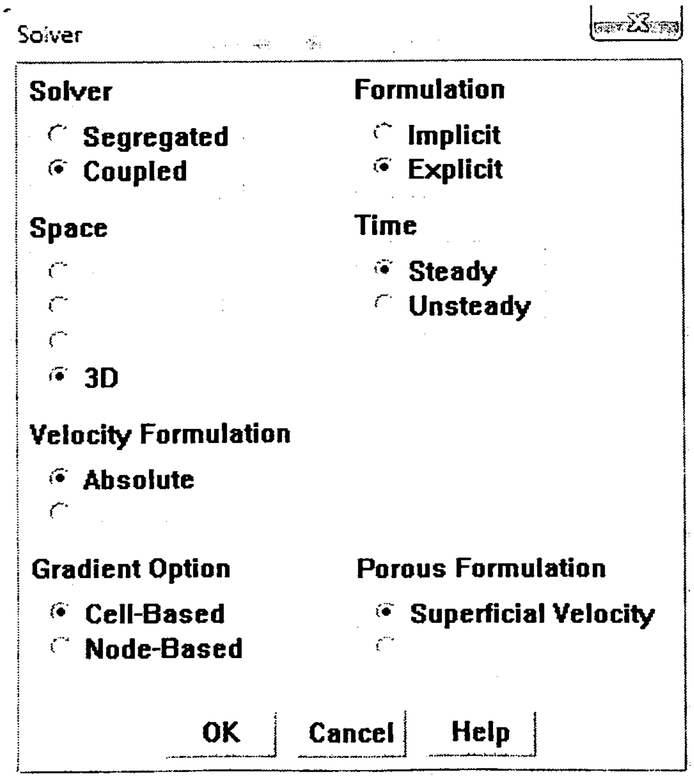

Figure 3.3: Solver

Reynolds number is:

Length of aircraft (l) = 180mm = 0.18m

V_∞ =-295

ρ = 1.176

$\mu = 1.7894*10^{-5}$

$$Re = \frac{\rho V_{\infty} l}{\mu} = 3.5 \times 10^{6}$$

Hence consider the turbulent flow. So to capture this flow use Spalart-Allmaras.

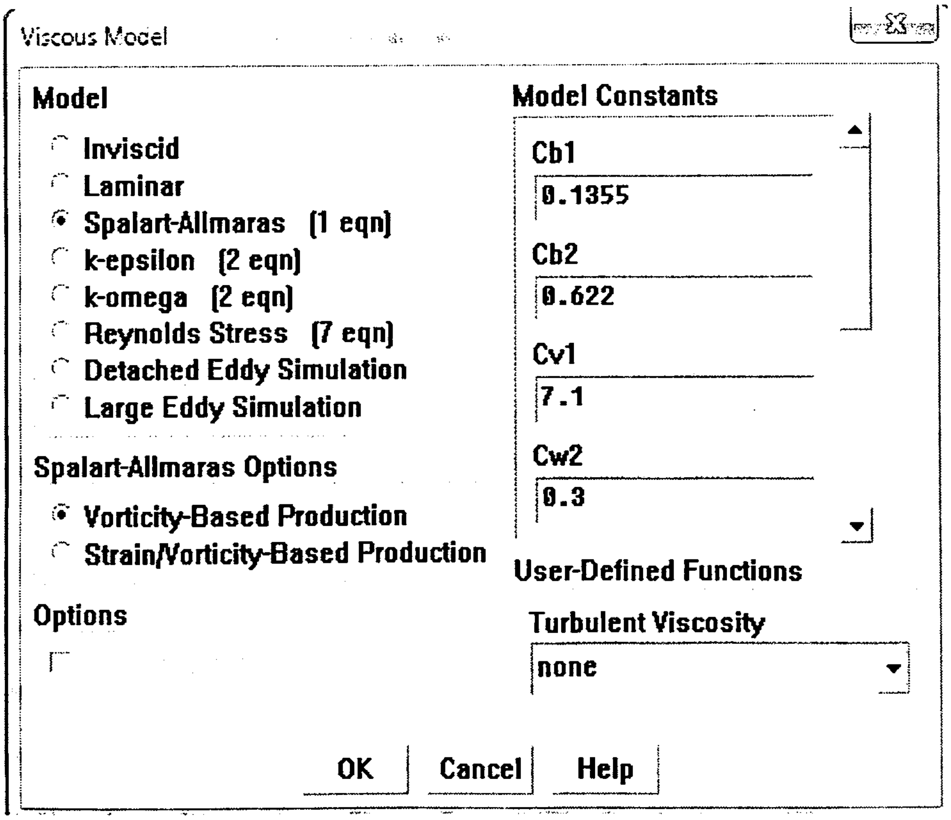

Figure 3.4: Viscous Model

To consider the ideal gas and hence the temperature, energy equations are required.

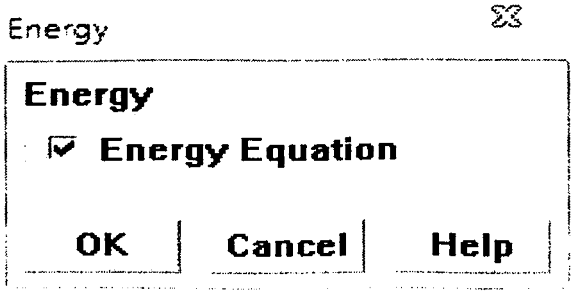

Figure 3.5: Energy Equation

Pressure far field is the pressure field surrounding the aircraft which was set at 0.85 mach (high subsonic speed) at the ambient temperature of 300 K these conditions chosen in accordance with the 747 flying conditions as a subsonic aircraft . With the aircraft proceeds in z-direction, the flow velocity is registers negative z-direction. The turbulent viscosity ratio was assumed to be 2.

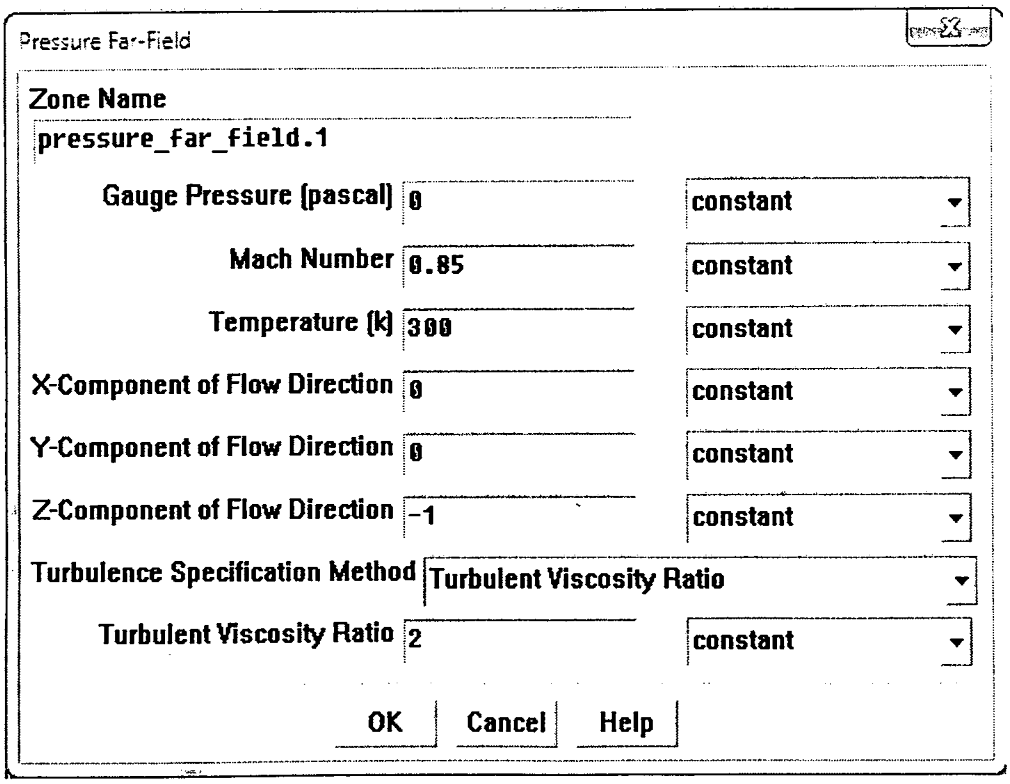

Figure 3.6: Pressure Farfield

The material properties have been specified. To get the temperature and viscosity relations consider sutherland viscosity.

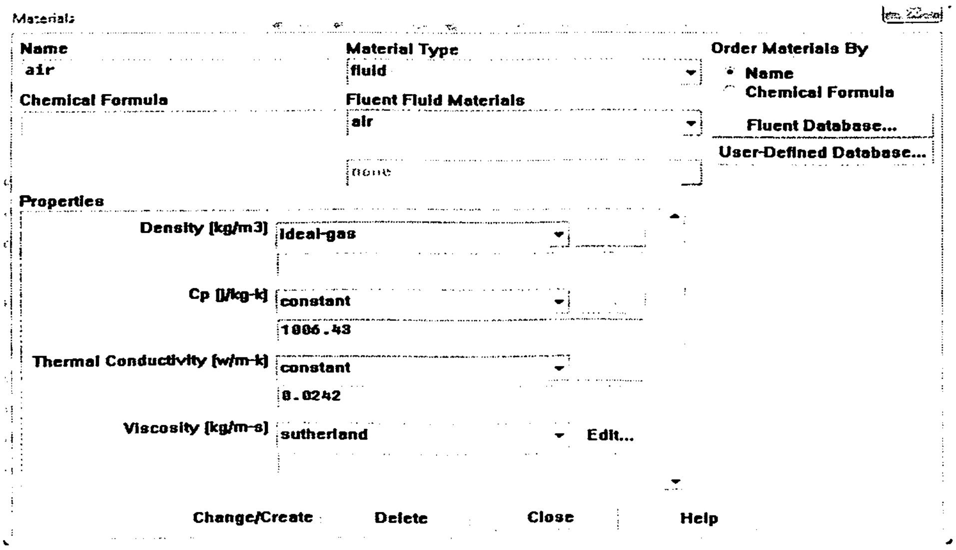

Figure 3.7: Materials

In the solver parameters, the courant number is slightly less than 1(0.9) for the explicit method. The discretization process is in the first order upwind for the higher rate of convergence but for the better results in future higher second order upwind options will be considered.

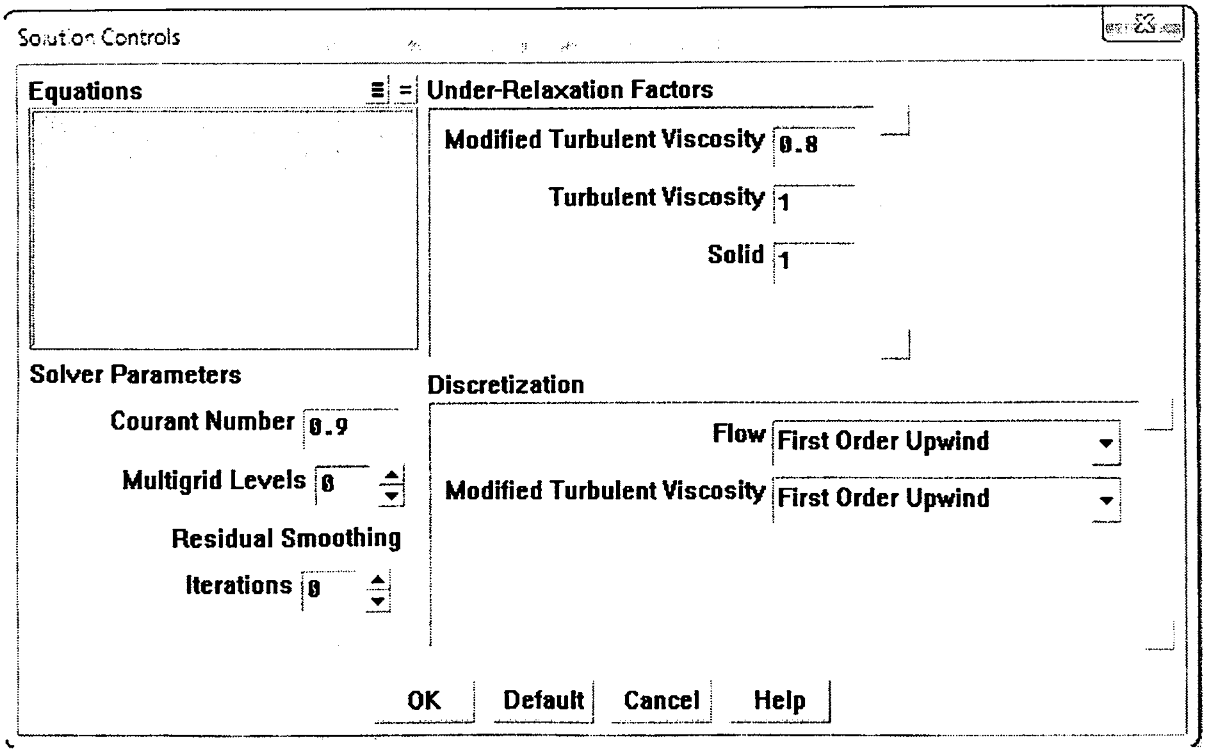

Figure 3.8: Solution Control

The initial values for the solution initialization have been calculated on the basis of pressure farfield. The fluent will automatically calculate the flow velocity according to the mach number, operating temperature, and the flow direction. Since the gauge pressure is the difference between absolute pressure and operating pressure. And in this analysis value of operating pressure is 101325 Pa. Therefore the gauge pressure will become zero.

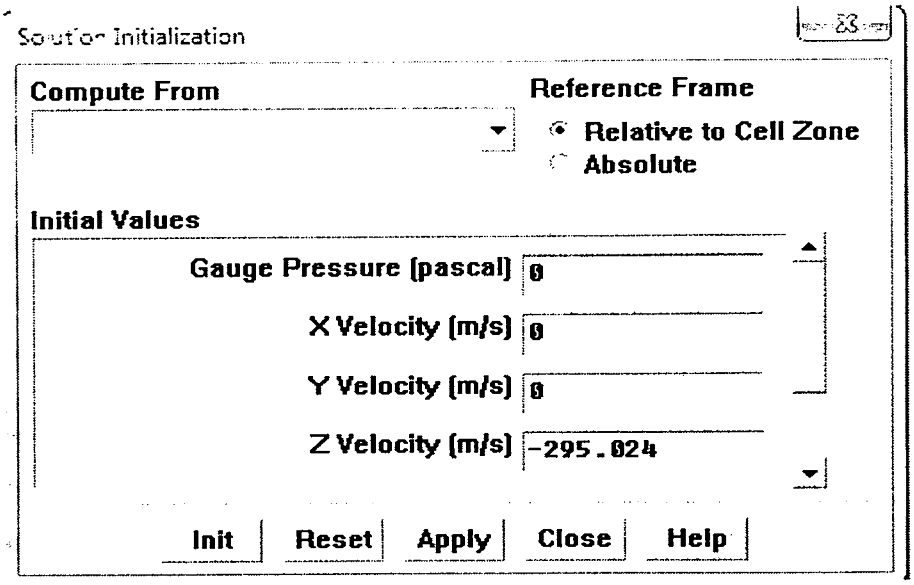

Figure 3.9: Solution Initilization

To get the residual plot the constraints are to be specified so the iterations of 1000 are selected and to get the converging trends the time limit is reduced to 0.001.

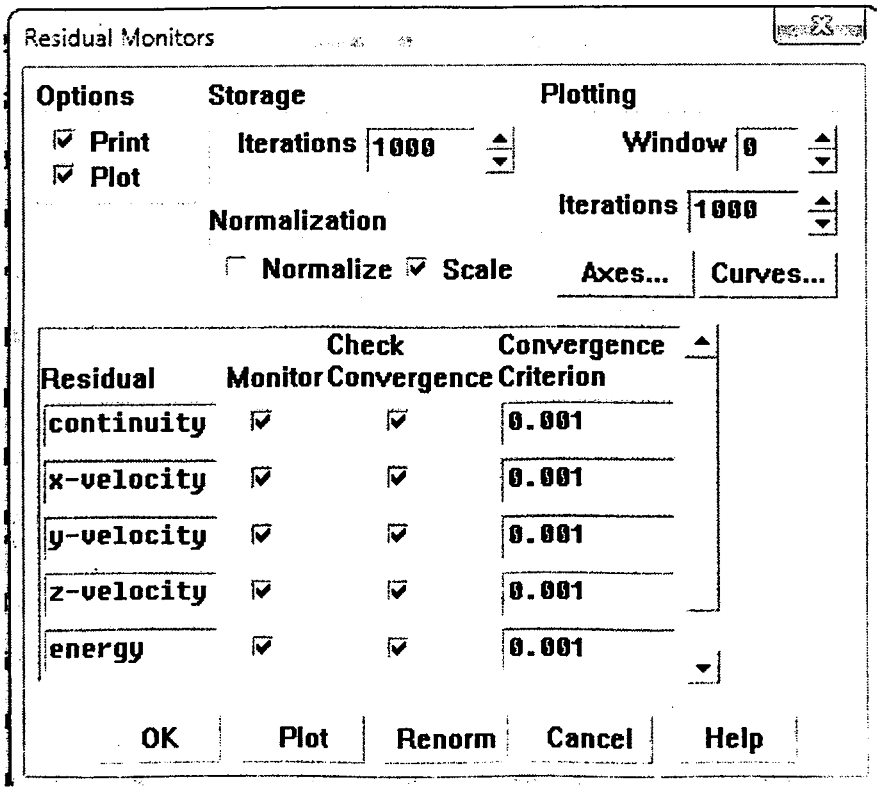

Figure 3.10: Residuals

3.4 Results

The graph depicts that our solution would converge with more iterations which would further reduce the residual

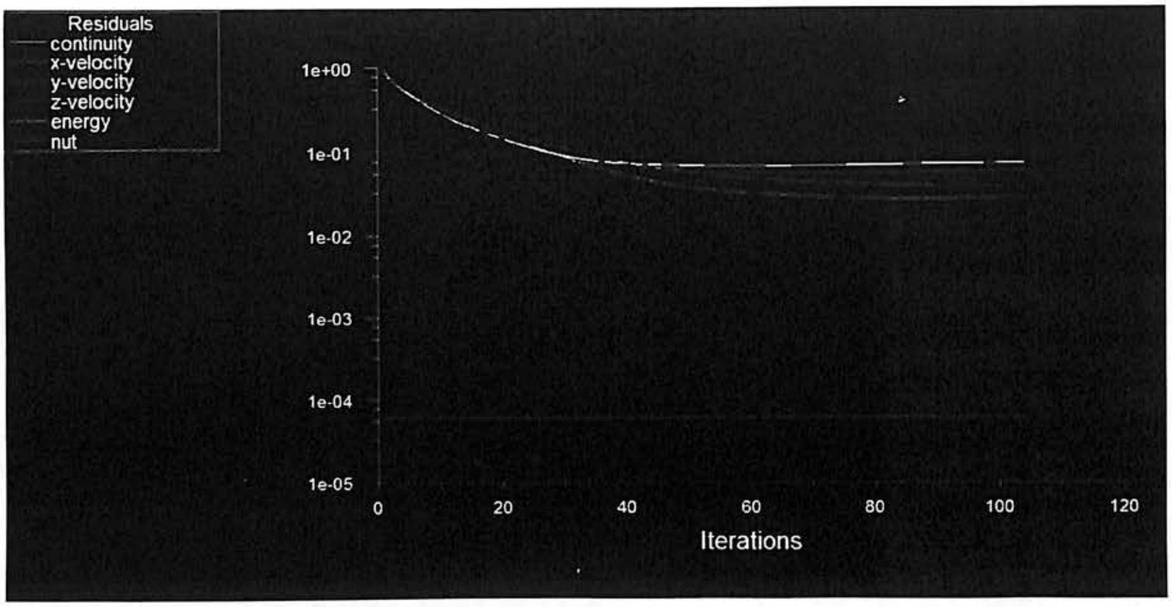

Figure 3.11:Scaled Residual

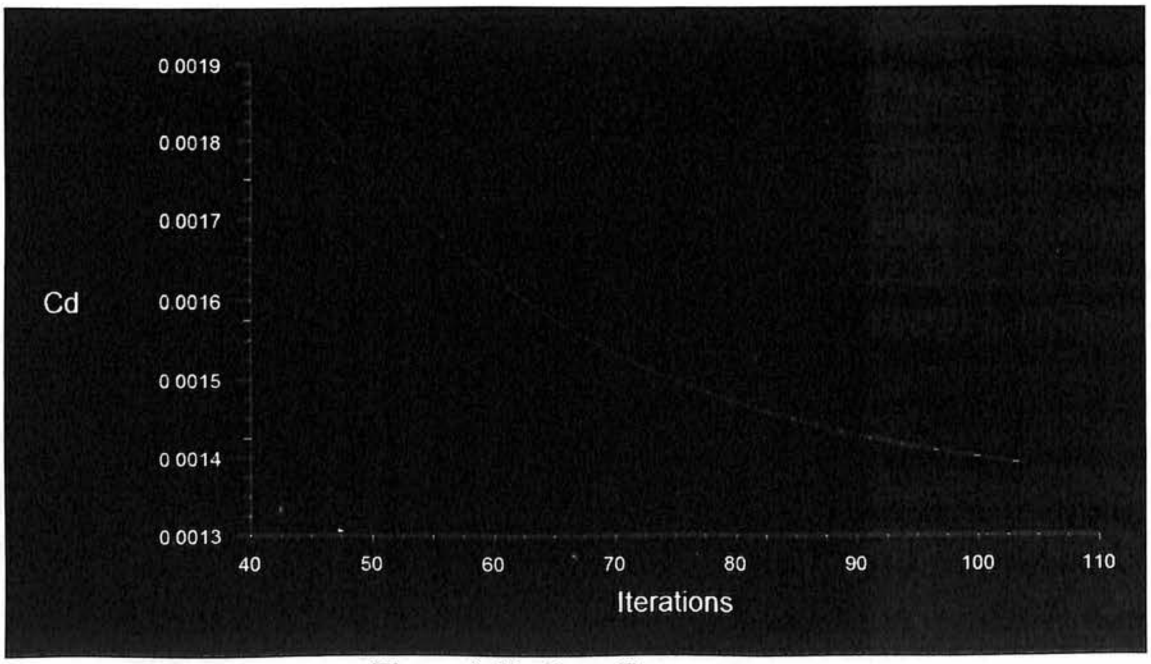

Figure 3.12: Drag Convergence

Graph showing the falling coefficient of drag with increasing number of iterations.

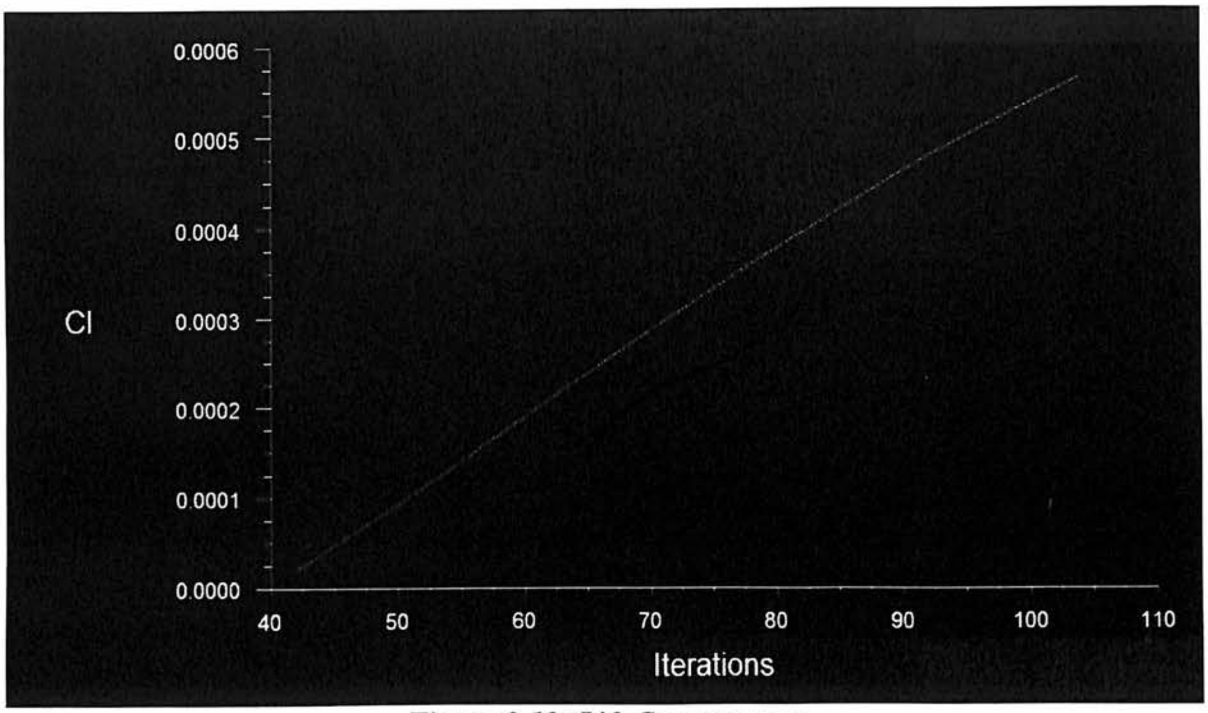

Figure 3.13: Lift Convergence

This graph shows the liner rise in the values of the lift. But since it is a symmetric aerofoil, the initial values of lift are zero at zero angle of attack and any value there is because of the fuselage shape.

In the following three graphs the hot zones of absolute pressure, static pressure and temperatures are at the stagnation points on the leading edges and front sections. With maximum absolute pressure of $1.72*10^5$ Pascals at the wing leading edges and then gradually falls as it approaches the trailing edge. Same is the case with the temperature that increases in correspondence with the pressure energy over the aircraft surface and is maximum at wing leading edges of magnitude of $3.56 *10^2$ Kelvin.

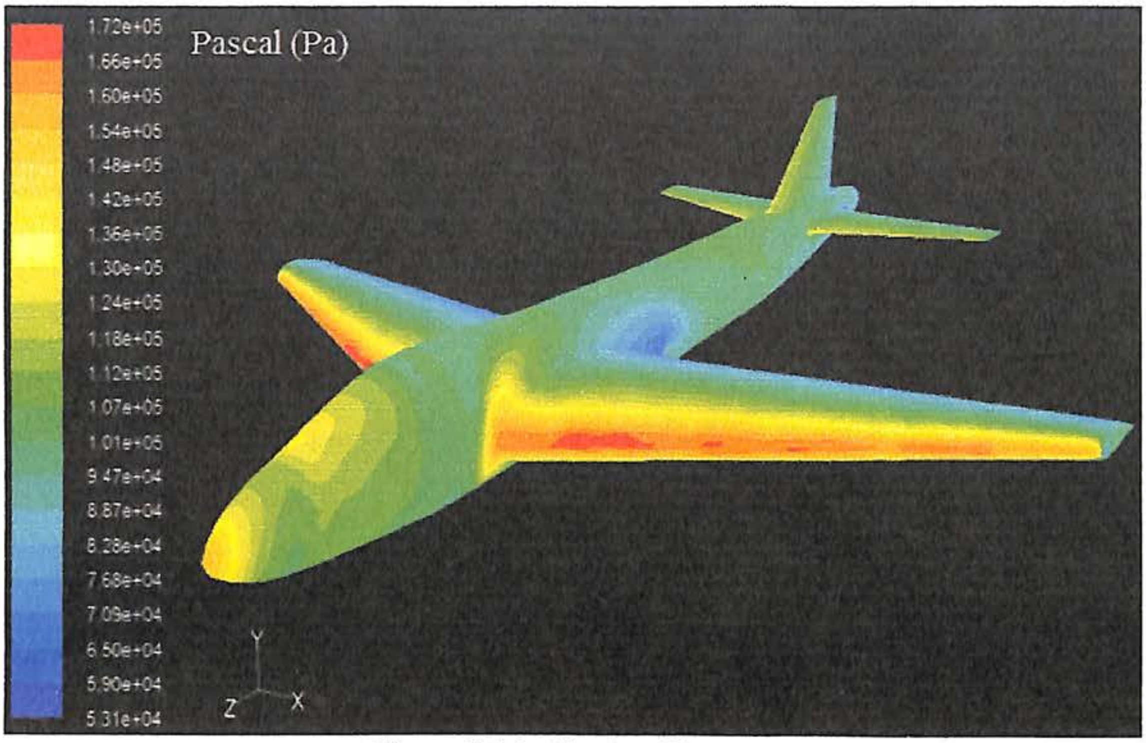

Figure 3.14: Absolute Pressure

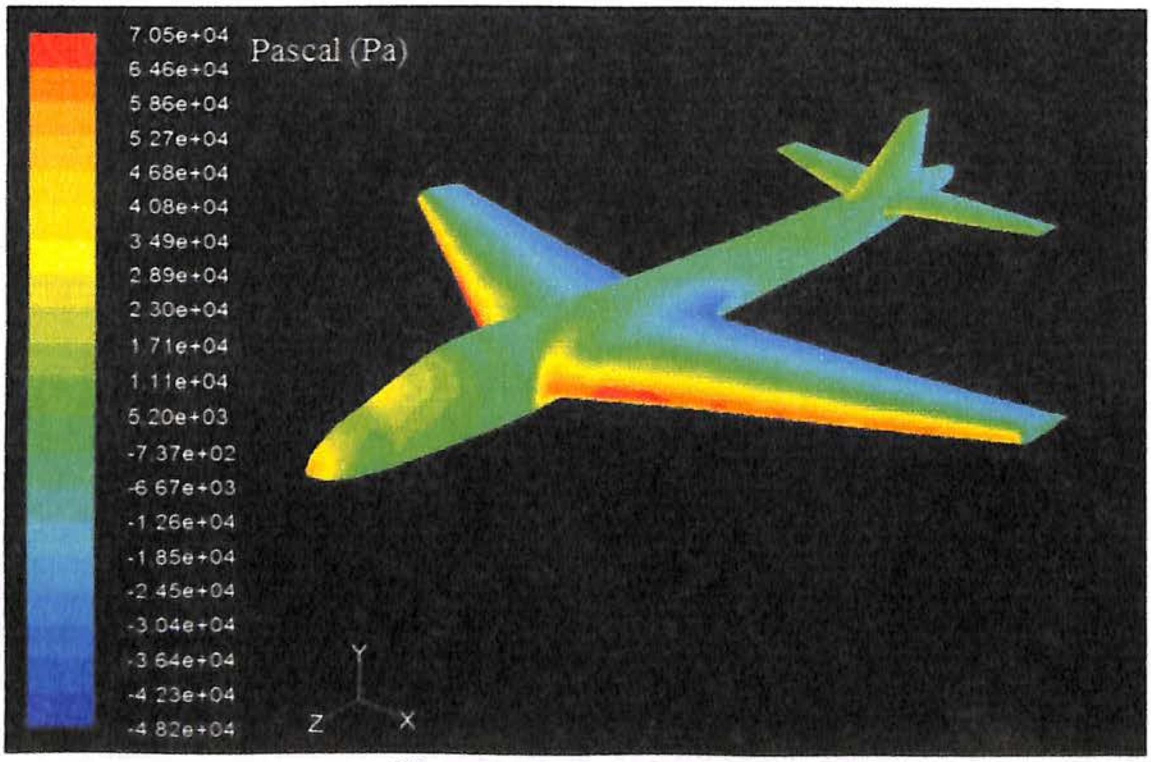

Figure 3.15: Static Pressure

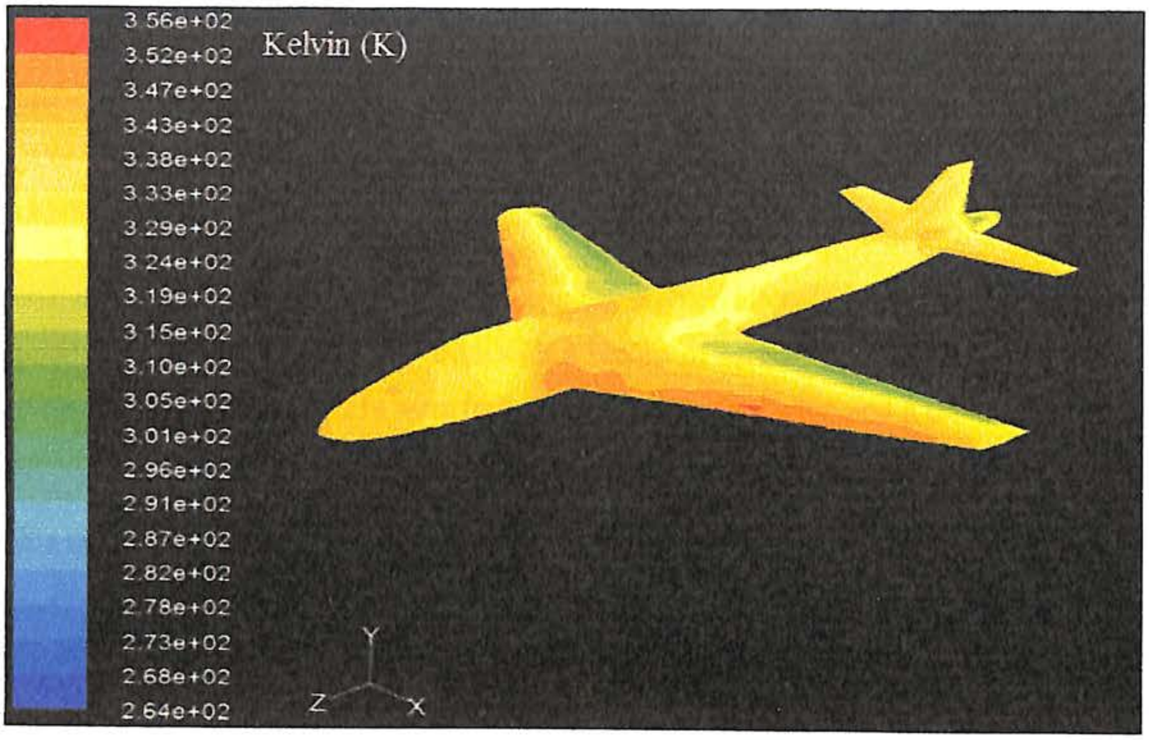

Figure 3.16: Temperature

CHAPTER 4: FINAL MODEL

4.1 Introduction

MiG 21 has been the soviet bloc's forerunner fighter aircraft that represents a transition period of air force of many countries with a record of serving more than 40 nations on the planet. Some 11,000 MiG 21s were built, mostly in Soviet Union and a few hundred in India and Czechoslovakia.

This delta wing plane has 57° sweep angle at the leading edge and a dihedral angle of 2° .It has a semi-monocoque construction. With a cone covering the air inlet duct to engines, that has three positions for different mach numbers to facilitate an unhindered air flow while tackling shockwaves, this aeroplane can reach speeds upwards of 2 Mach. The tail section of the aircraft has a vertical stabilizer with a sweep angle of 60° and a horizontal stabilizer with a sweep angle of 57°.

4.2 Modelling

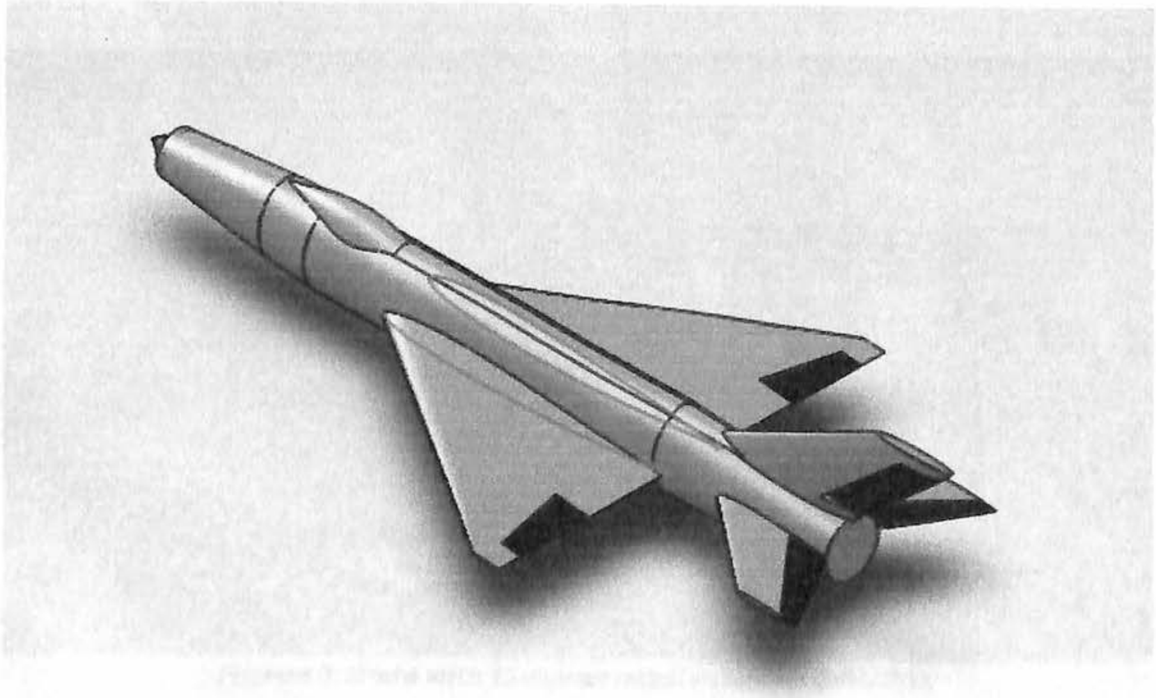

Figure 4.1: MiG 21 Model

Models are made in SolidWorks in a single piece, i.e. without using any assemblies. For every deflection new model was generated and only one control surface is deflected at a time for the purpose and the other control surfaces are kept neutral.

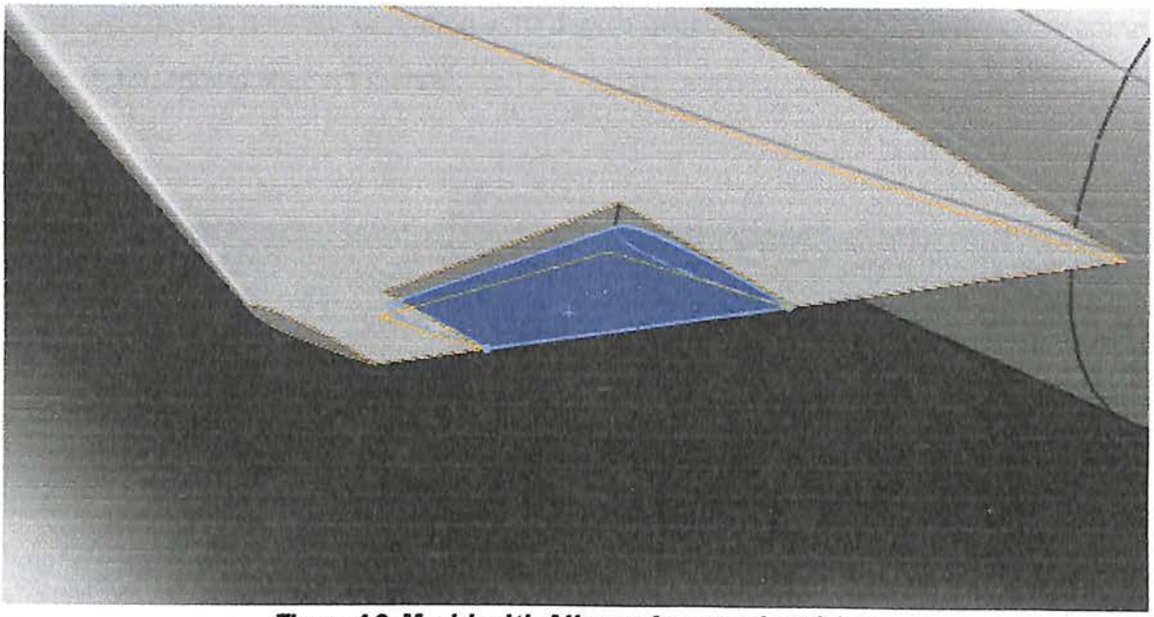

Figure 4.2: Model with Ailerons in neutral position

Ailerons is this model are kept inneutral position and some other control surface is deflected. Similarly, when any one control surface is deflected the other control surfaces are kept neutral.

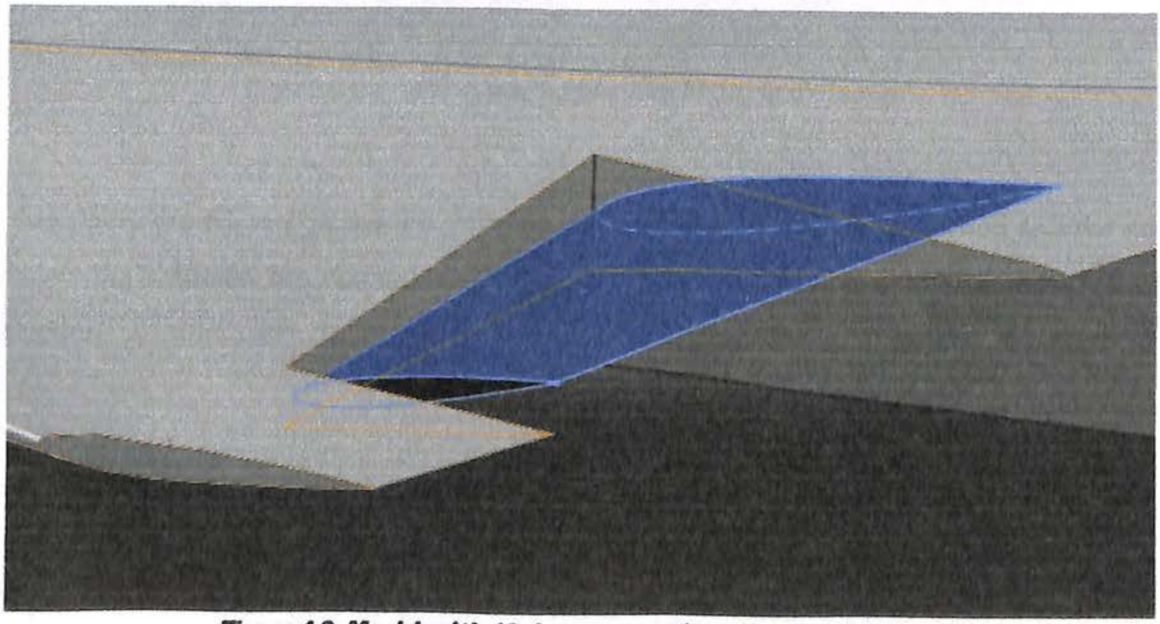

Figure 4.3: Model with 10 degrees negative aileron deflection.

This left aileron of the model has a deflection of 10 degrees in upward direction which means the other side that is not seen in this picture has a downward 10 degree deflection and together the ailerons here are providing a 10 degree negative deflection. All the other control surfaces in this model are kept neutral.

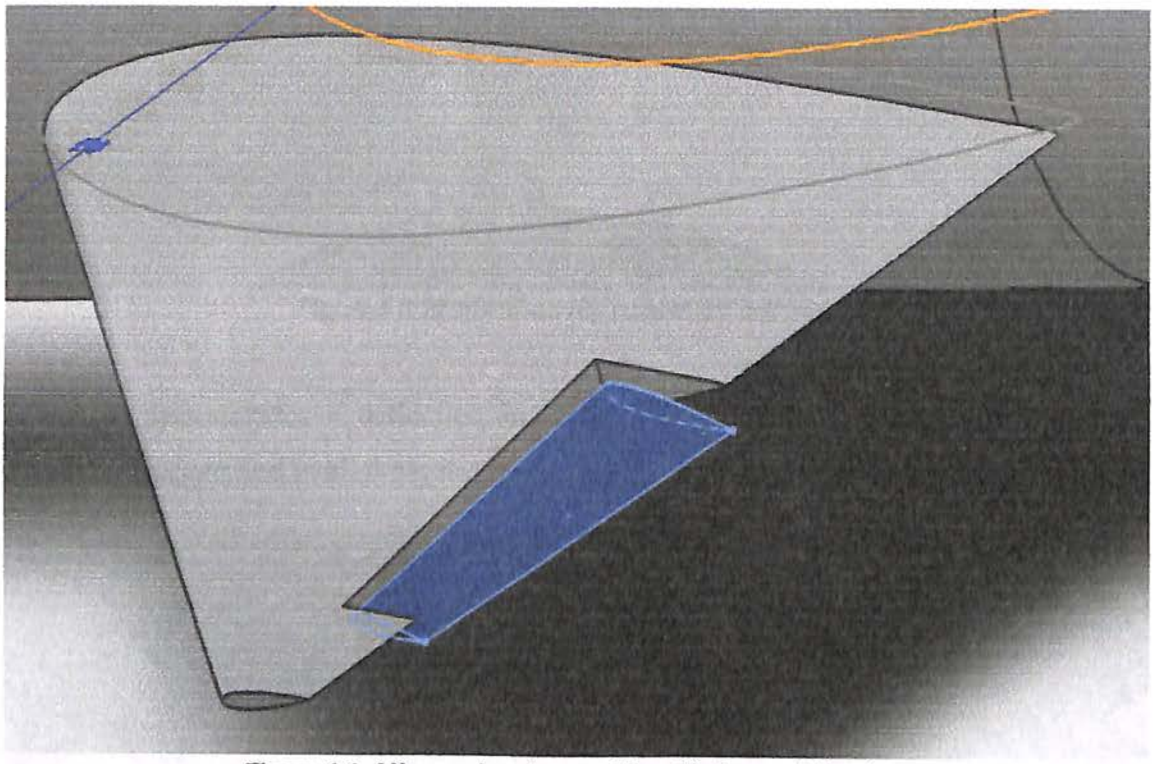

Figure 4.4: Aileron showing positive 10 degree deflection.

In this figure the aileron on the left wing has a downward 10 degree deflection and the non visible wing's aileron has upward 10 degree deflection for facilitating a 10 degree positive deflection.

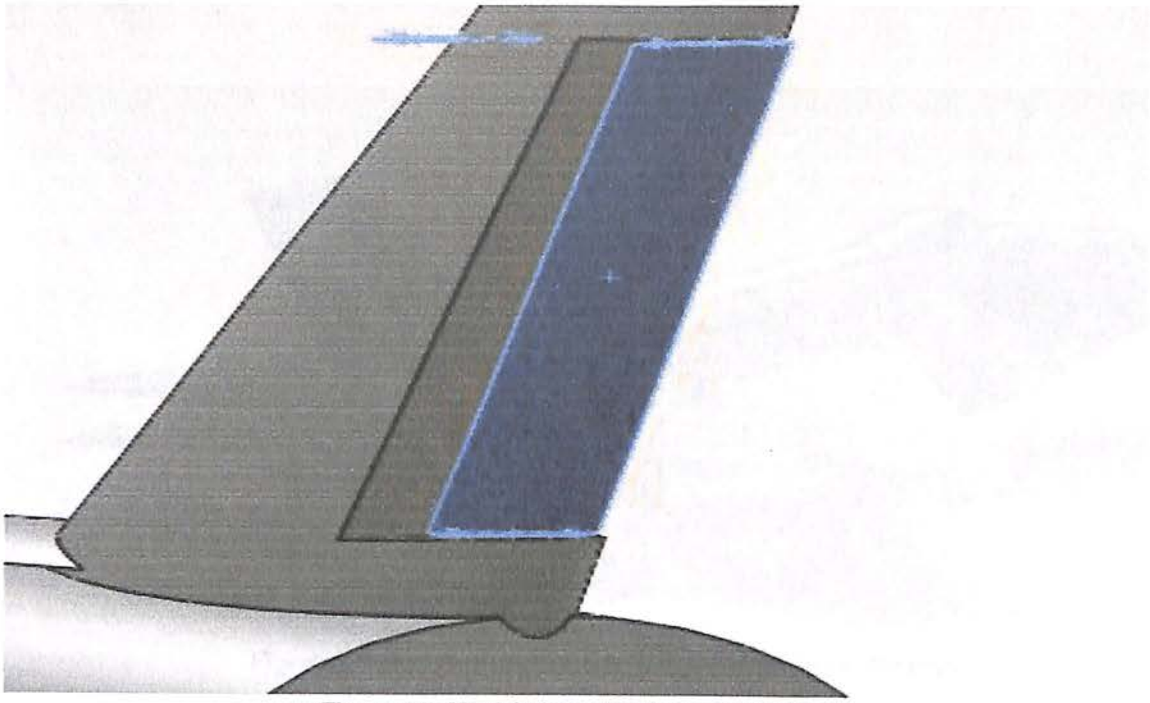

Figure 4.5: Model showing rudder deflection

This image demonstrates a deflection in rudder position. Three models were made for different deflections and in all these models the other control surfaces were kept neutral.

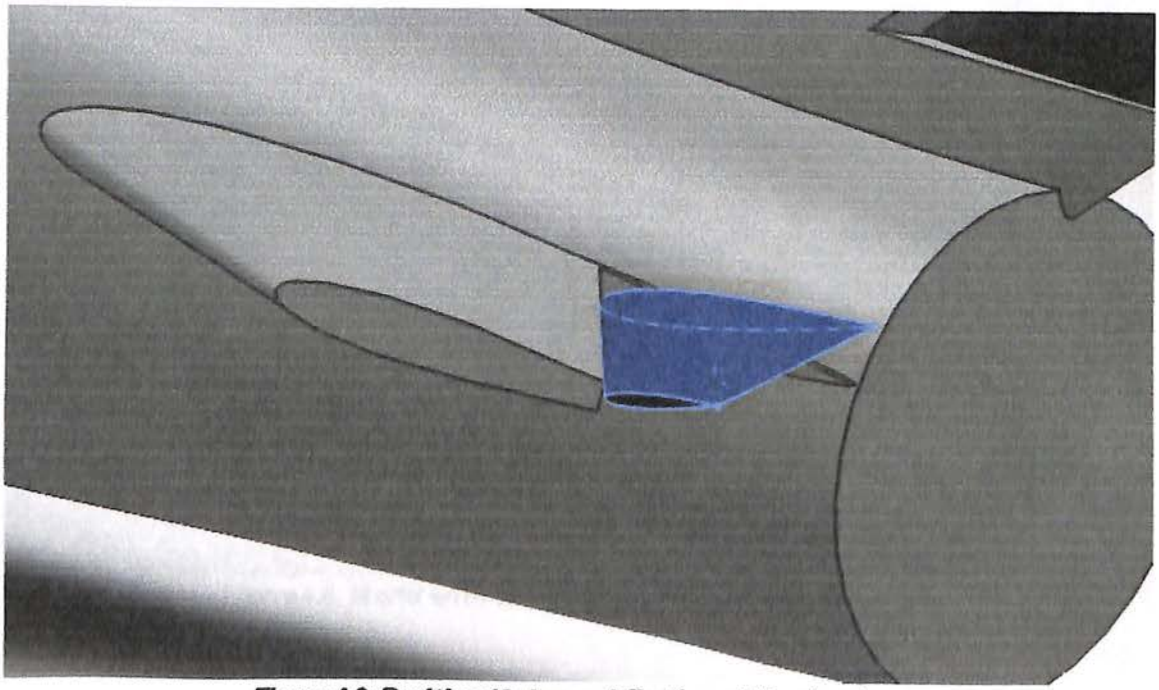

Figure 4.6: Positive 10 degree deflection of the elevator.

This image demonstrates a positive 10 degree deflection of the elevators, this was uniform on both sides and all the other control surfaces in the model were kept neutral.

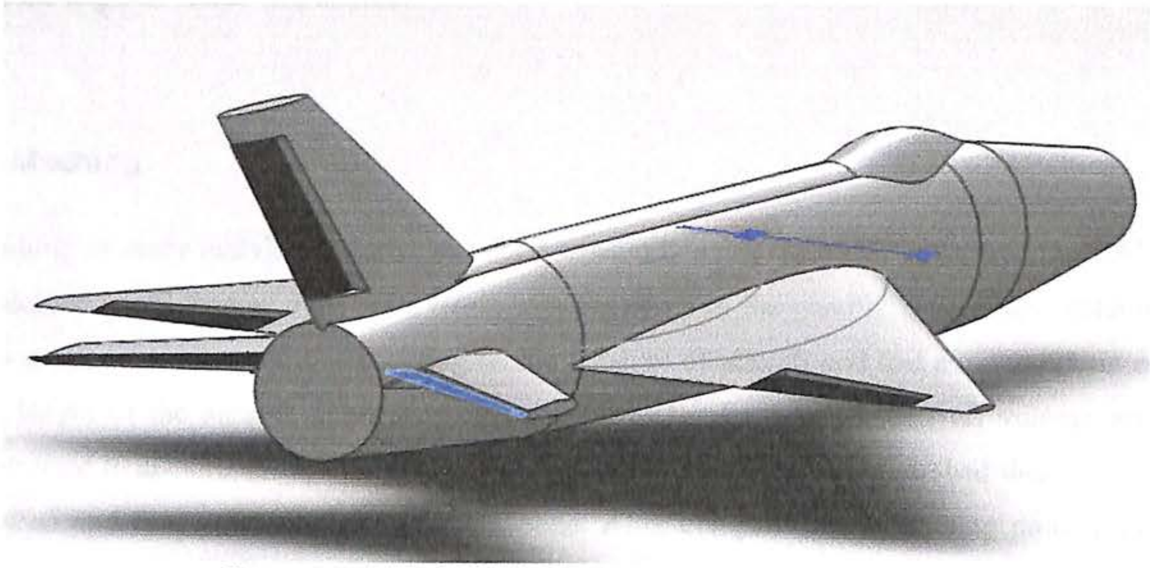

Figure 4.7: Model with positive 10 degree elevator deflection

This image shows the complete model which had a positive 10 degree elevator deflection while all the other control surfaces were kept neutral.

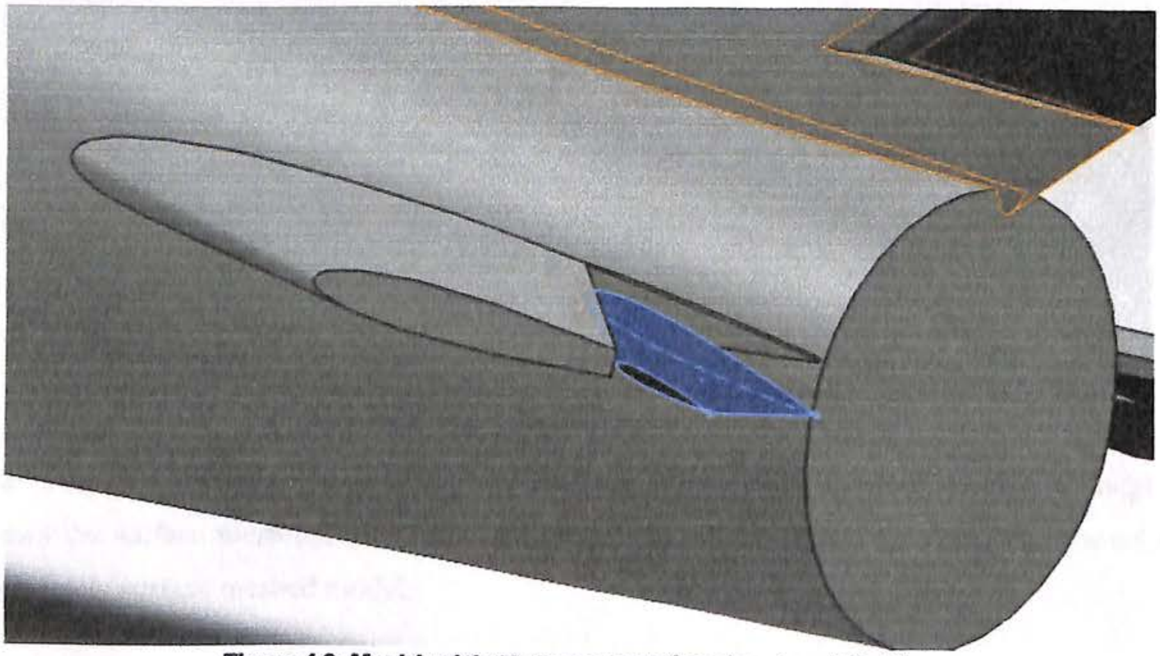

Figure 4.8: Model with 10 degree negative elevator deflection

This figure demonstrates a negative ten degree deflection in the elevator from the model which was made for negative 10 degree elevator deflection. The other elevator also is deflected 10 degrees downward and all the other control surfaces are kept neutral.

Similarly three other models were made which had all the control surfaces in neutral position and the angle of attack was altered.

4.3 Meshing

Meshing of every individual model was done separately using GAMBIT software. The CAD models in IGES format from SolidWorks was imported to the gambit. A spherical volume was created with its centre considering with the centre of aircraft and had a radius 25 times the length of the aircraft. From this volume, the real volume or the aircraft volume was subtracted to acquire a single volume. This final volume was first edge meshed then surface meshed and finally volume meshing was done. After completing the meshing process the boundary conditions were fed i.e. aircraft was taken as the wall and the sphere as the pressure farfield. Then this file was exported to fluent.

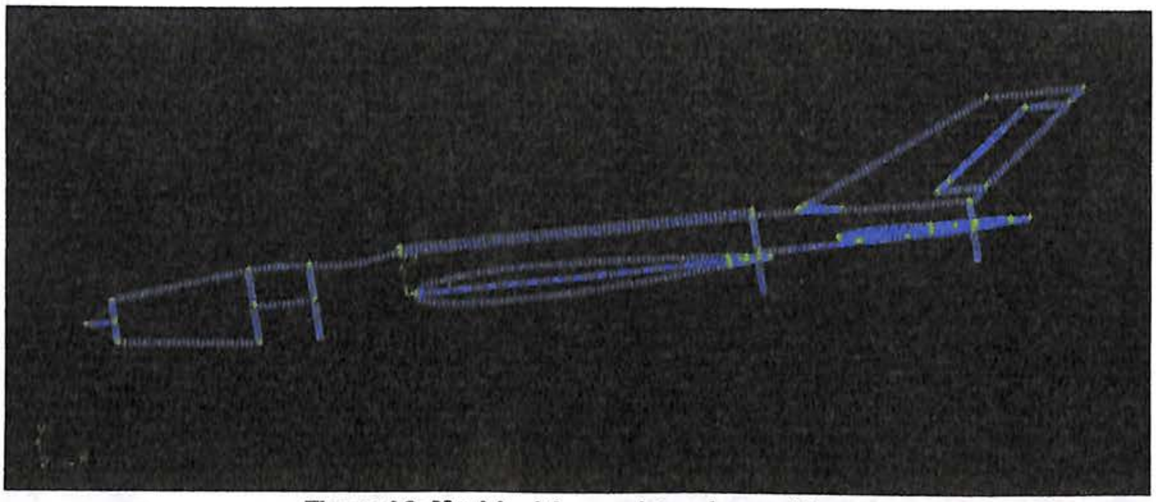

Figure 4.9: Model with complete edge meshing.

The image demonstrates the first stage of meshing, that is edge meshing, next two images shows the surface meshing. First one has wing surface meshed and the other one shows a completely surface meshed model.

Figure 4.10: Surface meshed wing

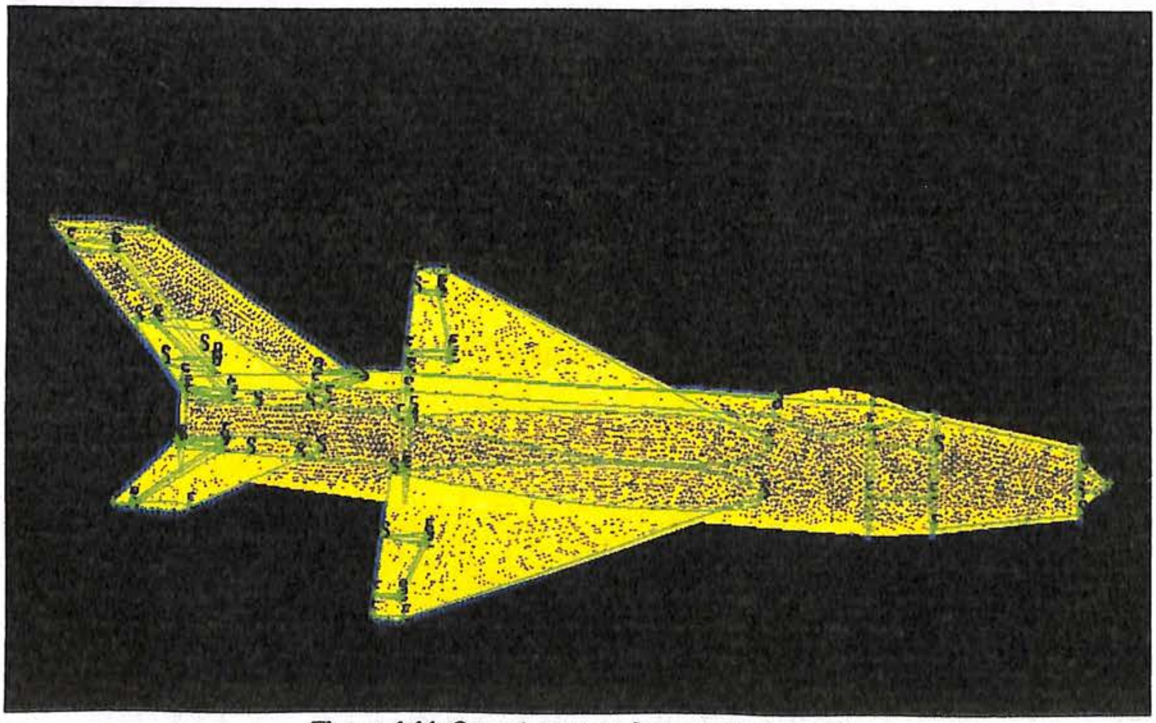

Figure 4.11: Completely surface meshed model

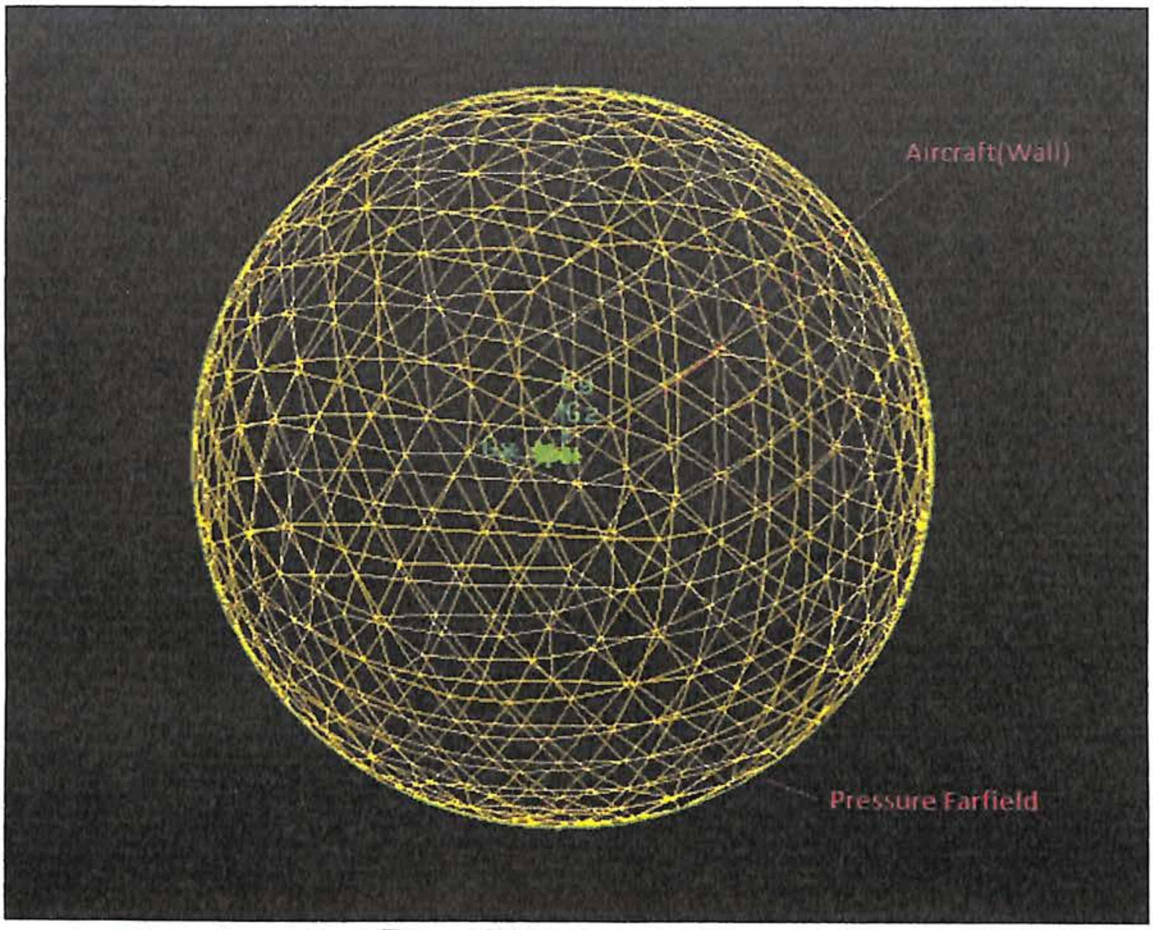

Figure 4.12:Volume meshed model

This image demonstrates volume meshing of the model. Volume meshing is done after the model is surface meshed.This was completely meshed and re-meshed until there were no errors encountered. After completing the meshing process the boundary conditions were fed i.e. aircraft was taken as the wall and the sphere as the pressure farfield. Then this file was exported to fluent.

4.4 Solver

To solve the problem FLUENT software is used. The first step is to read the case and have a grid check for any problem in the mesh. The following options have been used to solve the problem:

- Coupled Solver: It is used for compressible flow. It is known from experience that a coupled solver compute better results for compressible flow.
- Implicit Formulation: It is stable for larger time step, even with courant no. > 1 and is suitable for obtaining steady state solution but requires more memory whereas explicit formulation gives time accurate solutions for a unsteady problem and require low memory. Explicit is also unstable for courant no. > 1 and hence can only be used with small time steps.
- Steady Time: the problem is steady as boundary conditions do not vary and the mach number is also constant.
- Absolute Velocity Formulation: There is no reference or relative velocity as only aircraft moving not the air.
- Cell-based Gradient Option: This gives the centroidal value based solution whereas node-based option gives the each point value based solution.

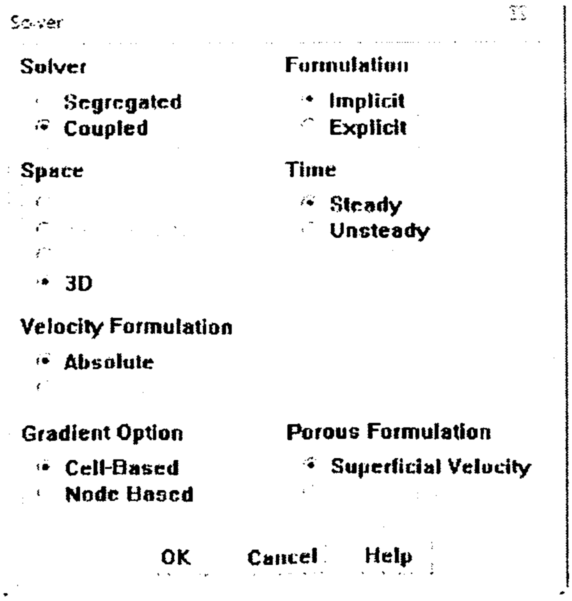

Figure 4.13: Solver

- Energy Equation: It is used for compressible flow and to consider the ideal gas as well as temperature variation. Since the problem specified has the mach of 0.5 so the compressible flow is considered.

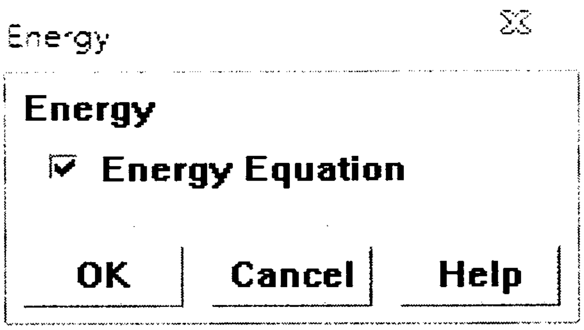

Figure 4.14: Energy Equation

- Spalart Allmaras (S-A) Turbulence Model: It is a one equation model used to capture the turbulence. This model is expected to produce faster results as compared to k-epsilon and k-omega. Since S-A model uses wall functions, therefore it is better for coefficient of lift and drag, Hence spalart allmaras is used.

    - K-epsilon (κ-ε) and K-omega (κ-ω): These are among the most common turbulence models. These are two equation models, which means, these include two extra transport equations to represent the turbulent properties of the flow. This allows a two equation model to account for history effects like convection and diffusion of turbulent energy.

      The first transported variable is turbulent kinetic energy, κ. The second transported variable is the dissipation rate, ε or specific dissipation, ω. It is the variable that determines the dissipation of the turbulence, whereas the first variable, κ, determines the energy in the turbulence.

- Turbulence Viscosity: As intensity and length scale turbulence specification method has been used, turbulence viscosity is not required.

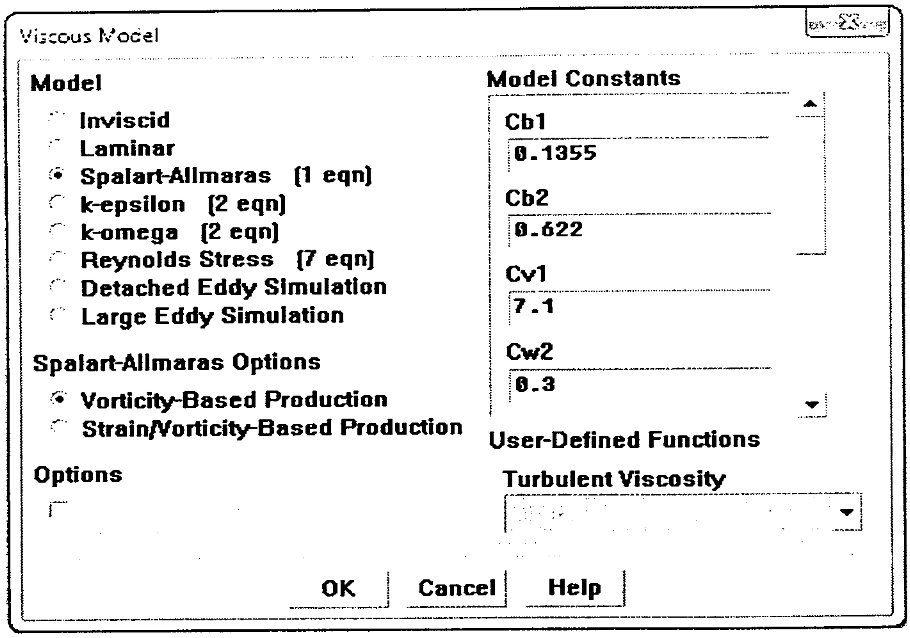

Figure 4.15: Viscous Model

- Ideal Gas: It is used to get the relation between pressure, velocity and temperature.
- Viscosity: Assumed to be constant as it the change is not too high. (To consider viscosity and temperature relation when change is high, sutherland is used.)

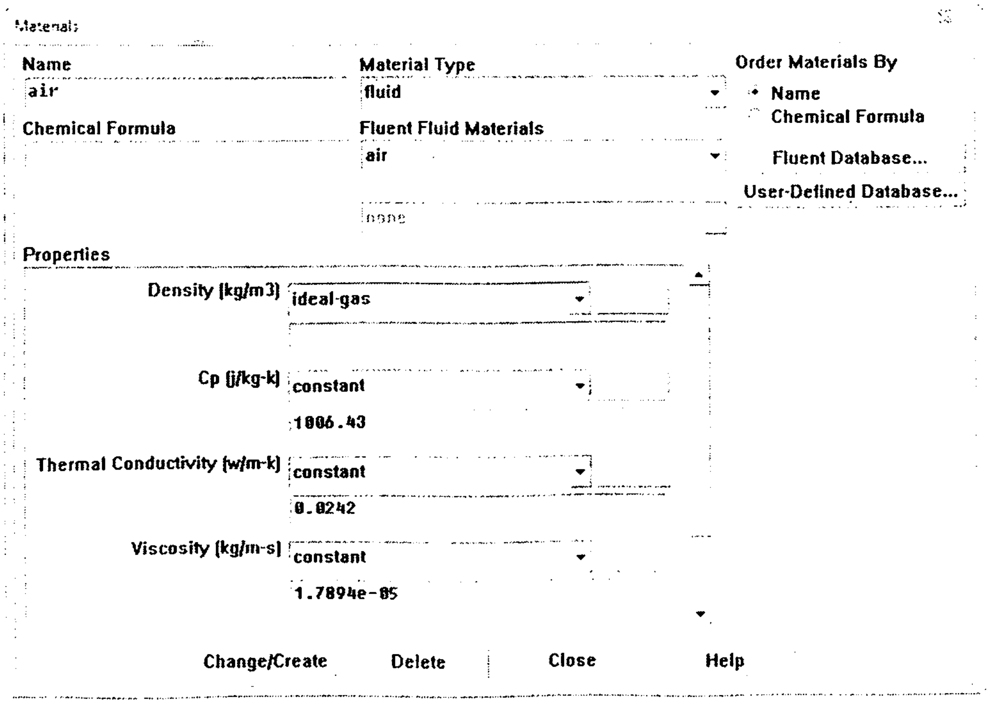

Figure 4.16: Materials

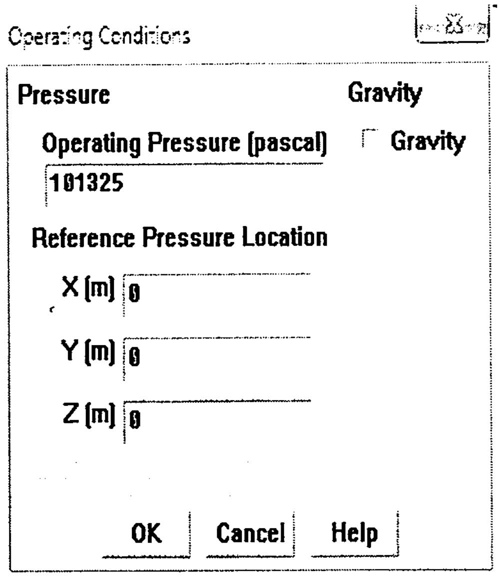

Figure 4.17: Operating Conditions

- The initial values for the solution initialization have been calculated on the basis of pressure far field.

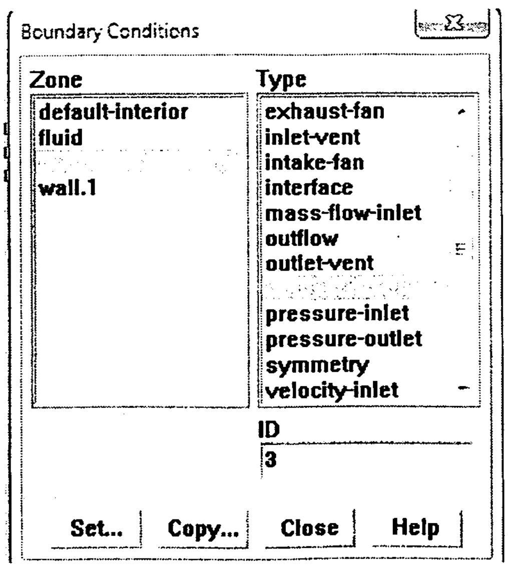

Figure 4.18: Boundary Condition

- Pressure far field is the pressure field surrounding the aircraft which is at 0.5 mach (low subsonic speed) and the ambient temperature of 300 K. As the aircraft is moving in the z-direction, the flow velocity is in the negative z-direction.

- Turbulence Specification Method: Intensity and scale method has been used. Turbulence intensity is the amount of turbulence existing already in the atmosphere (i.e. 1%) and turbulence length scale is the length up to which the turbulence penetrates.

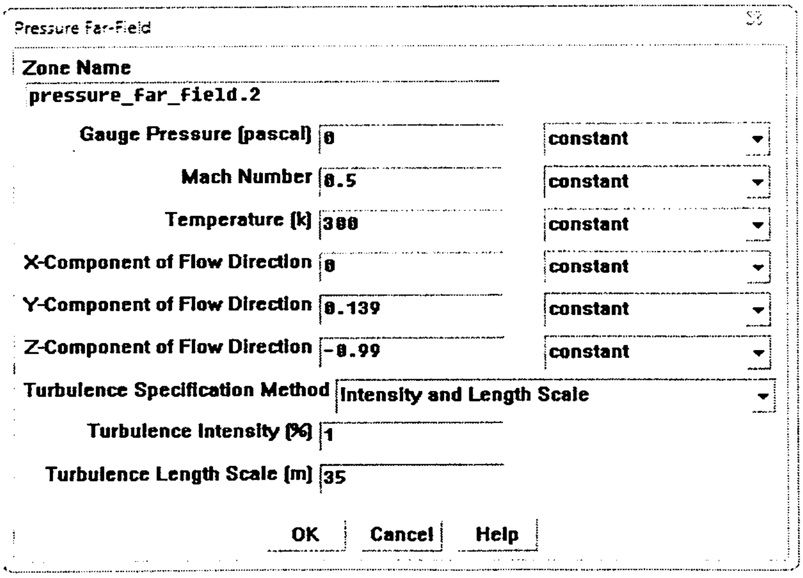

Figure 4.19: Pressure Far-Field

- First Order Upwind: It has higher rate of convergence and does not destabilize. Second order upwind gives better result which is more accurate as has less truncation error but require less courant number with same result. Therefore, it is advised to use second order upwind after completing first order.
- The Courant number is set to 2 as it is implicit formulation.

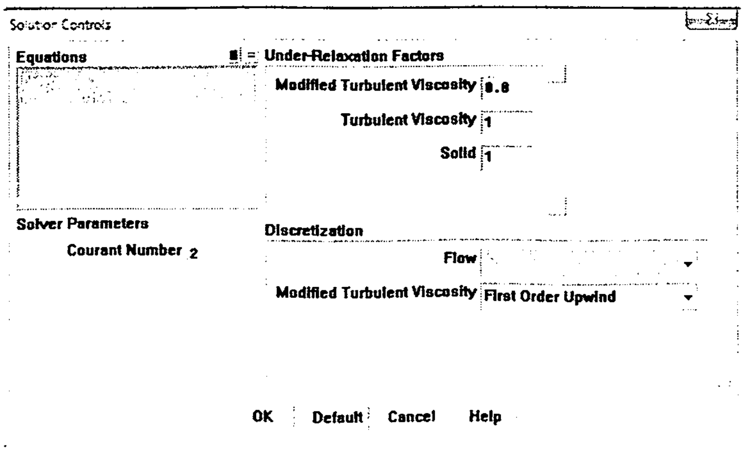

Figure 4.20: Solution Controls

- The solution is initialised and computed from pressure far field.

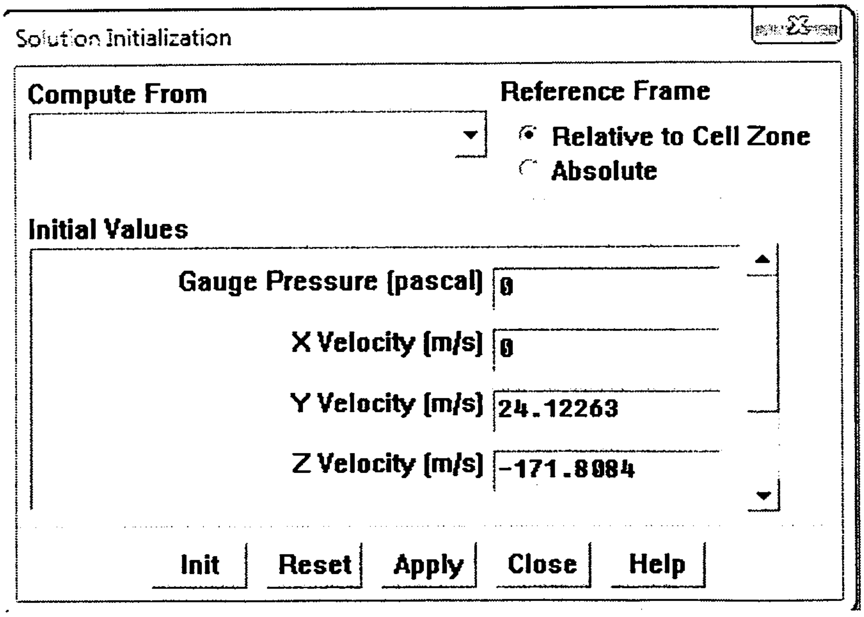

Figure 4.21: Solution Initialization

- In residual monitor convergence criterion is set to 10e-05.

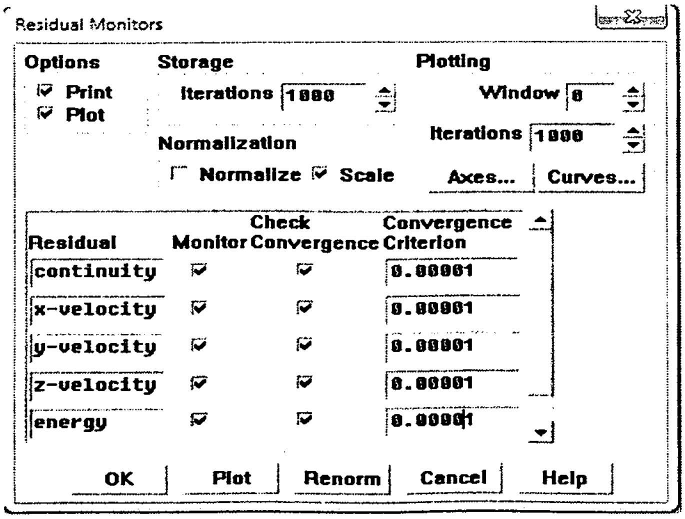

Figure 4.22: Residual Monitors

- In the force monitor the force vectors for drag and lift are specified:

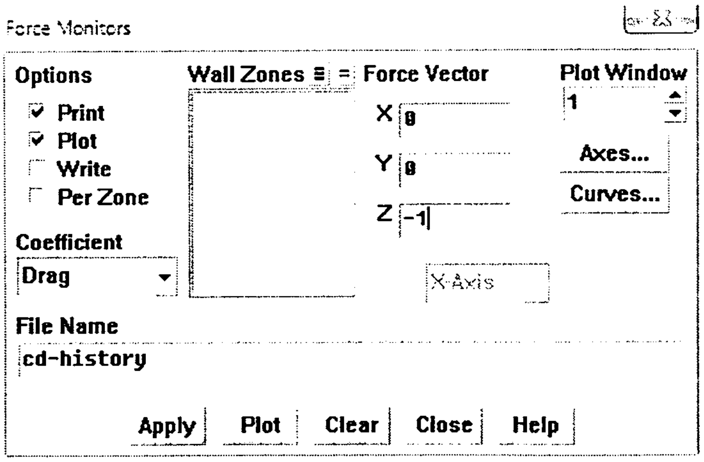

Figure 4.23: Force Monitors(drag)

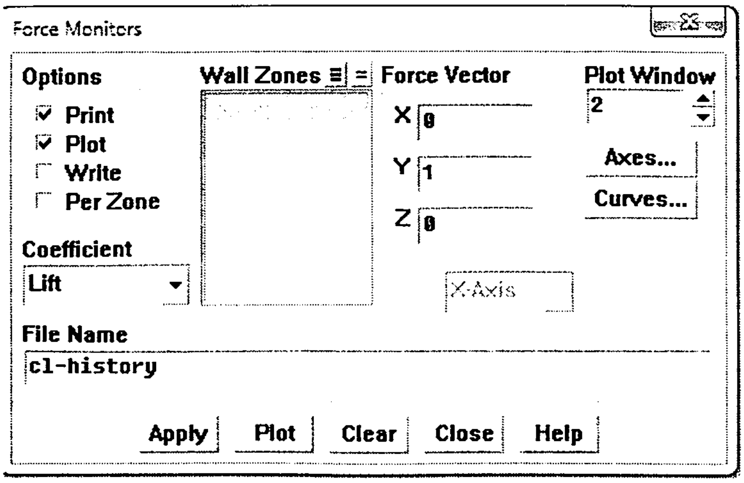

Figure 4.24: Force Monitors(lift)

- The reference values are used depending on the output or the result being calculated.
- Reference value is given as per pressure far field.

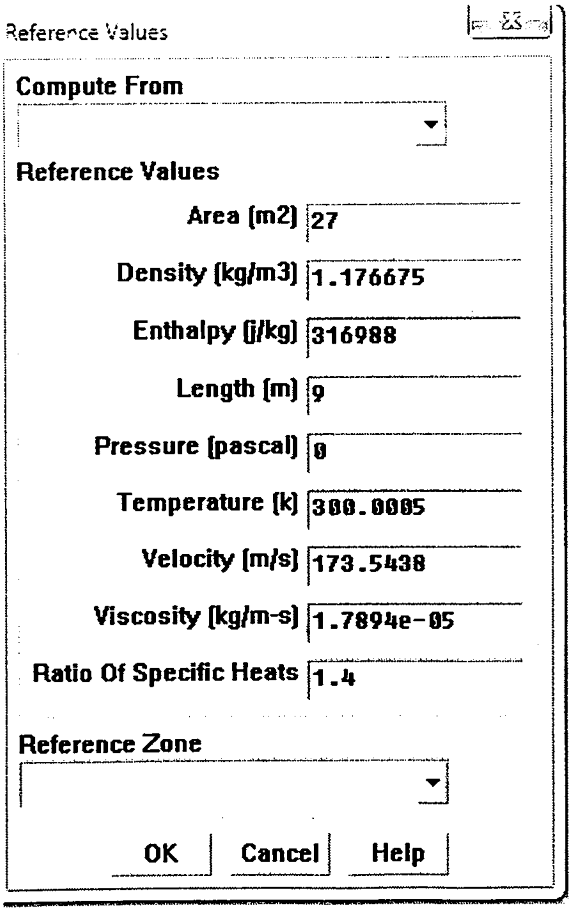

Figure 4.25: Reference Values

- The number of iterations were specified as 10,000 but the approximate number of iterations performed for convergence was about 6500-8500.

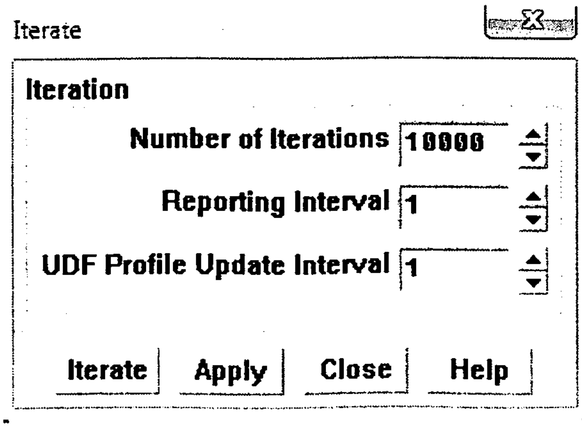

Figure 4.26: Iterate

4.4.1 ELEVATOR DEFLECTIONS

The following graphs are the end product of simulations that were done over the models for different angle of elevations.For elevator deflection of 10 degrees in upward direction:

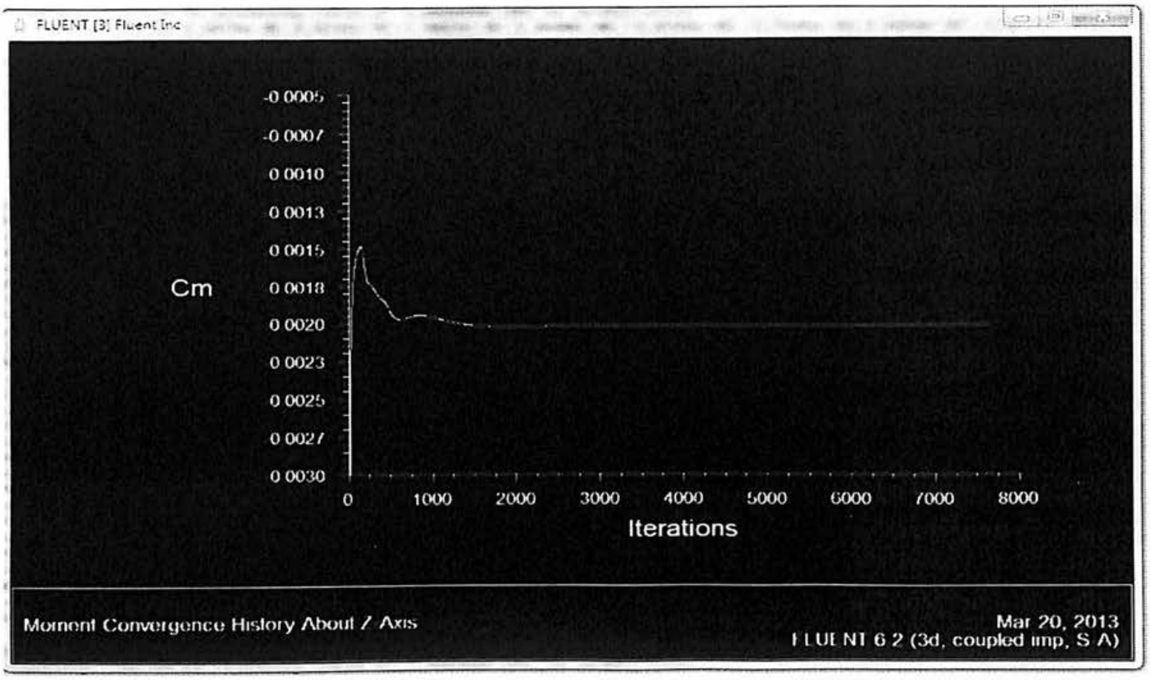

Figure 4.27: Graph showing variation in coefficient of moment with the number of iterations

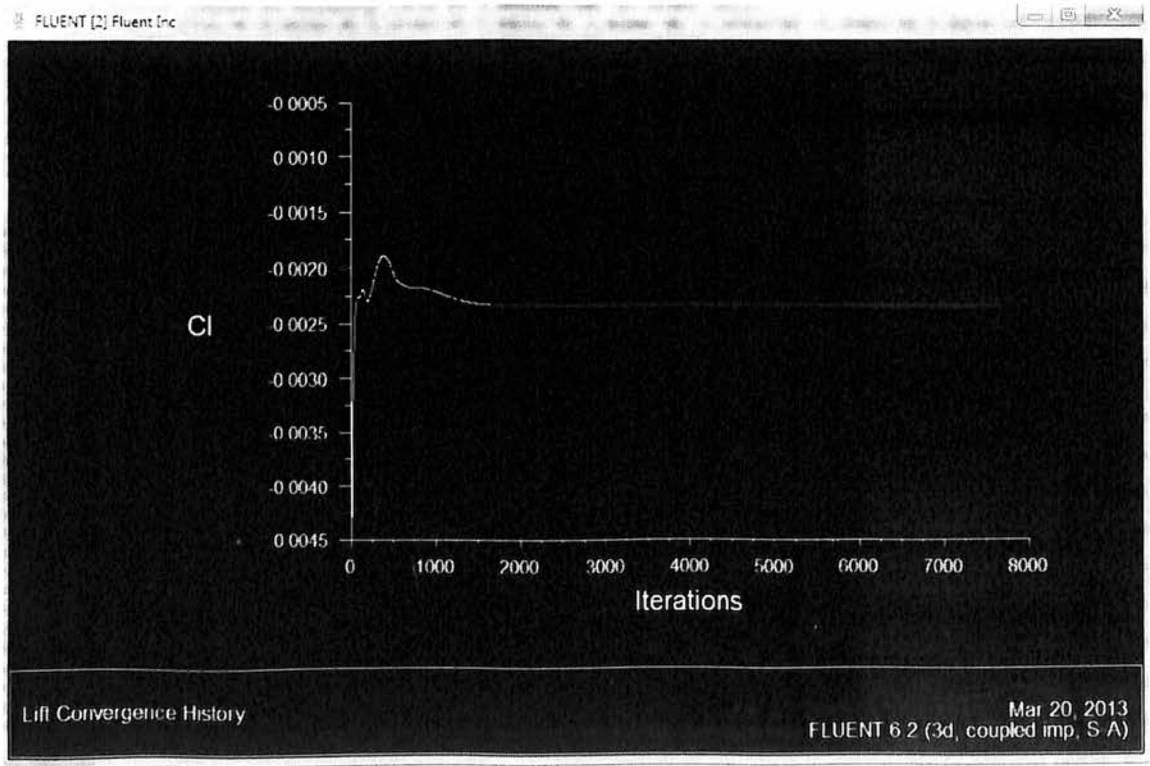

Figure 4.28: Graph showing variation of Coefficient of lift with the number of iterations

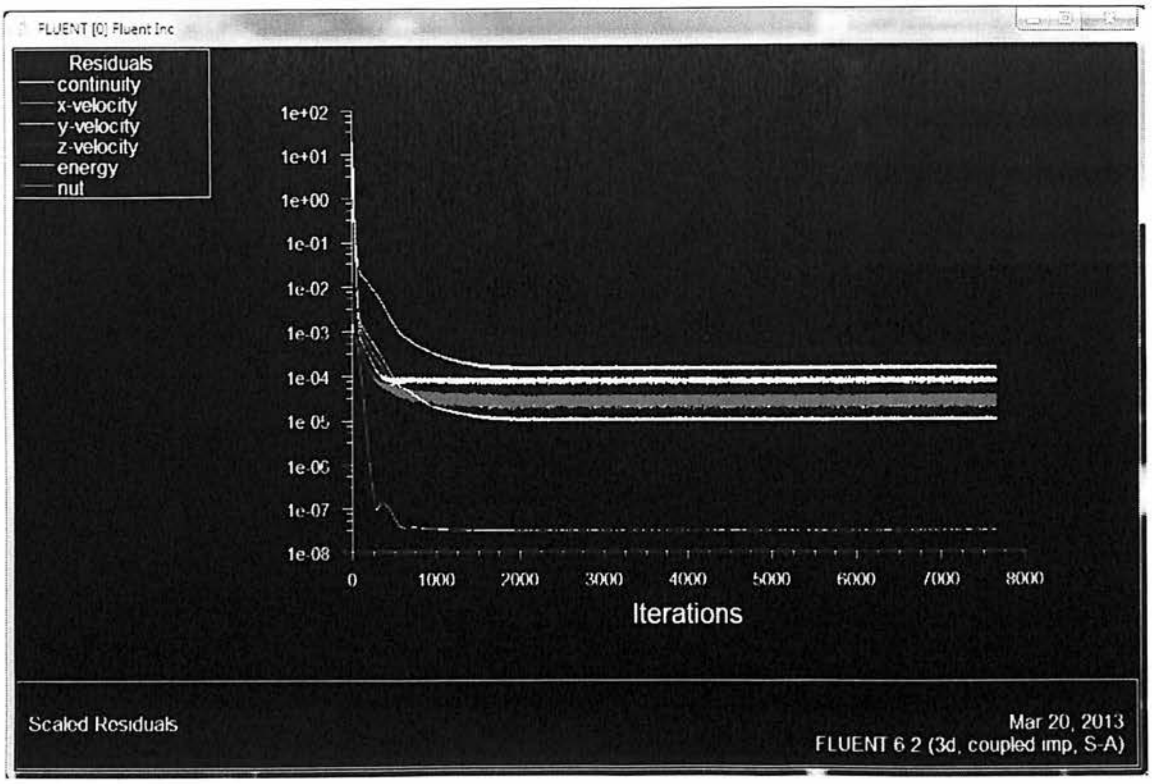

Figure 4.29:Graph showing variation of scaled residuals with number of iteration

For elevator deflection of 5 degree upward direction:

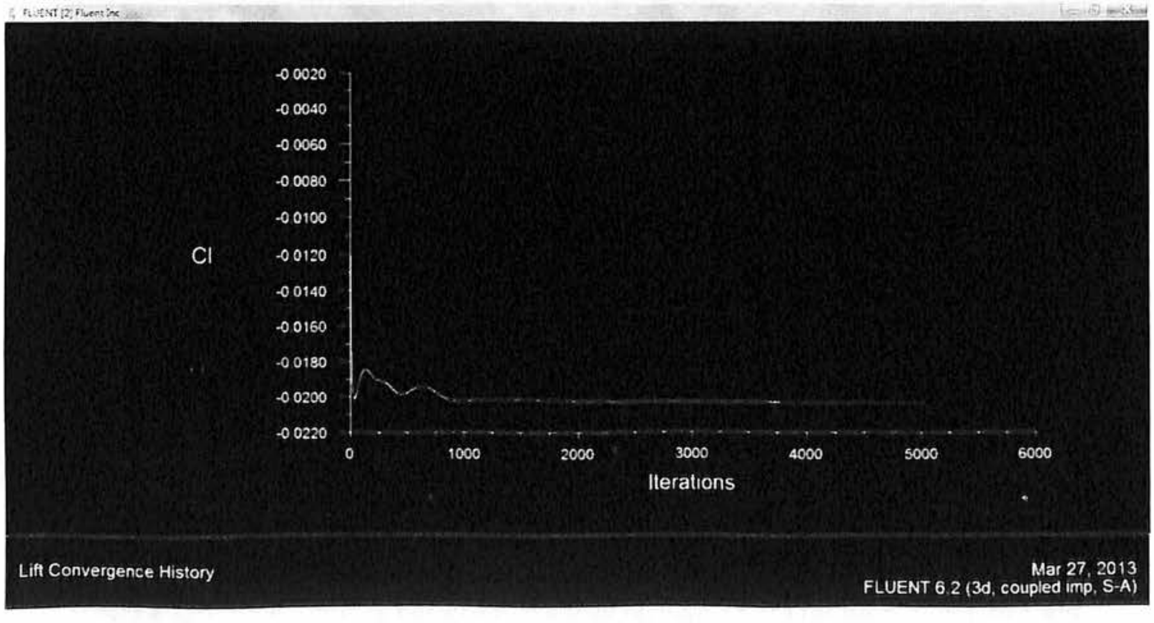

Figure 4.30:Graph showing variation of coefficient of lift with number of iterations

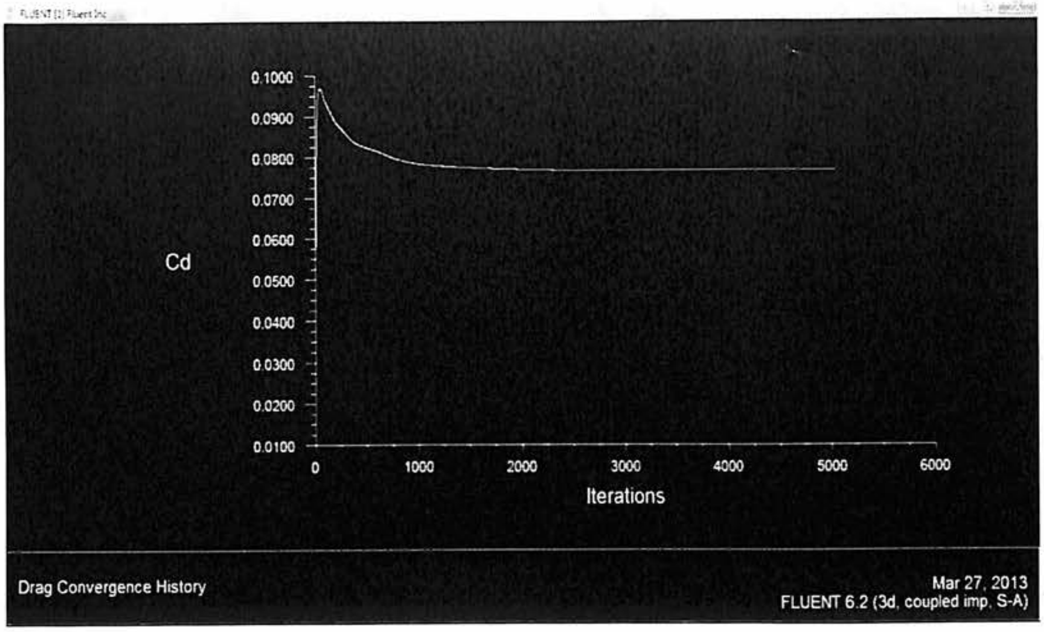

Figure 4.31: Graph showing variation of Coefficient with drag

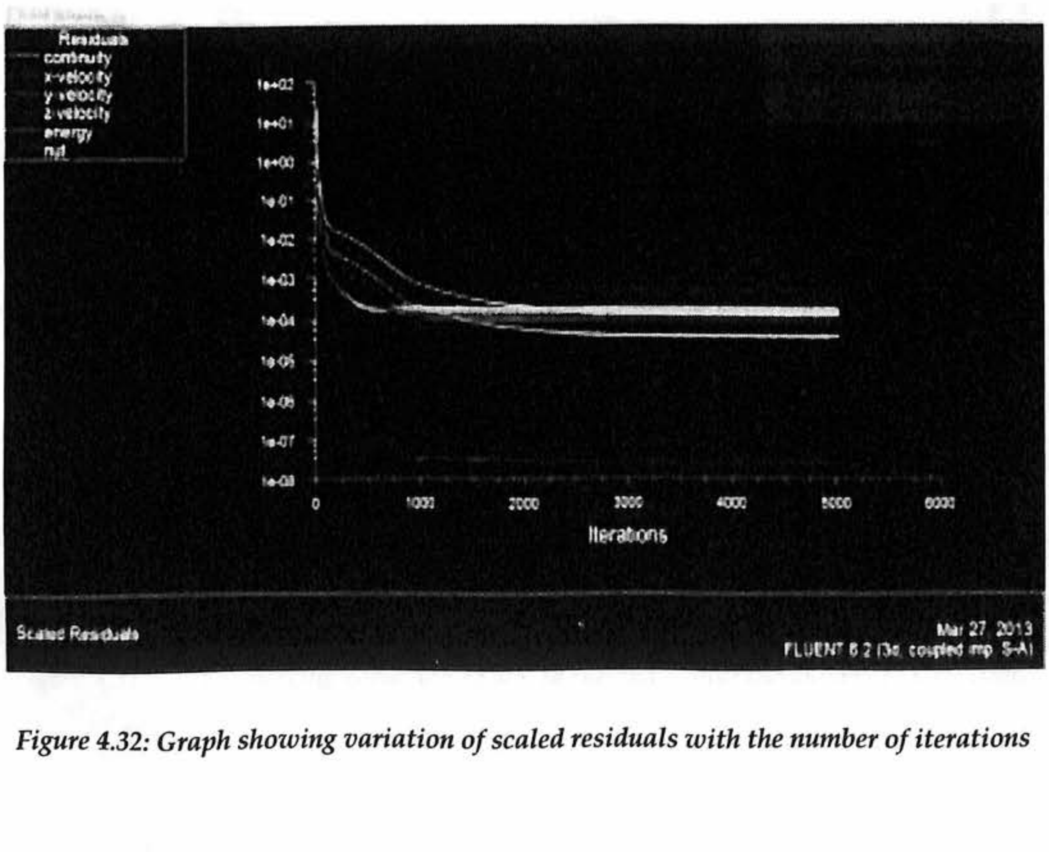

Figure 4.32: Graph showing variation of scaled residuals with the number of iterations

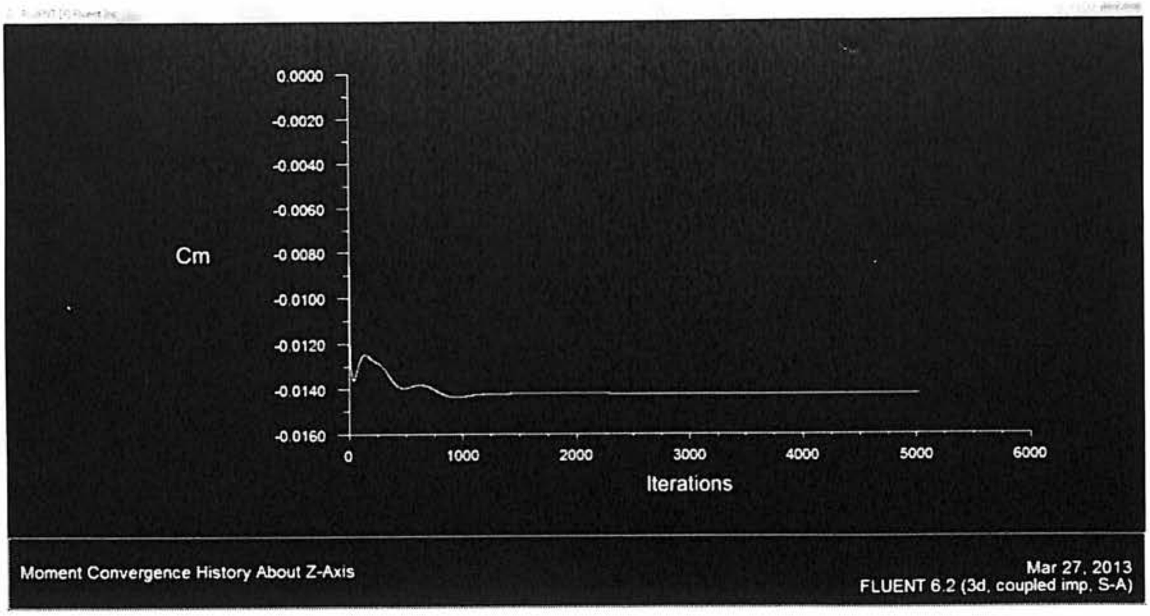

Figure 4.33:Graph showing variation of Cm with the number of iterations

For elevator deflection of 5 degree downwards:

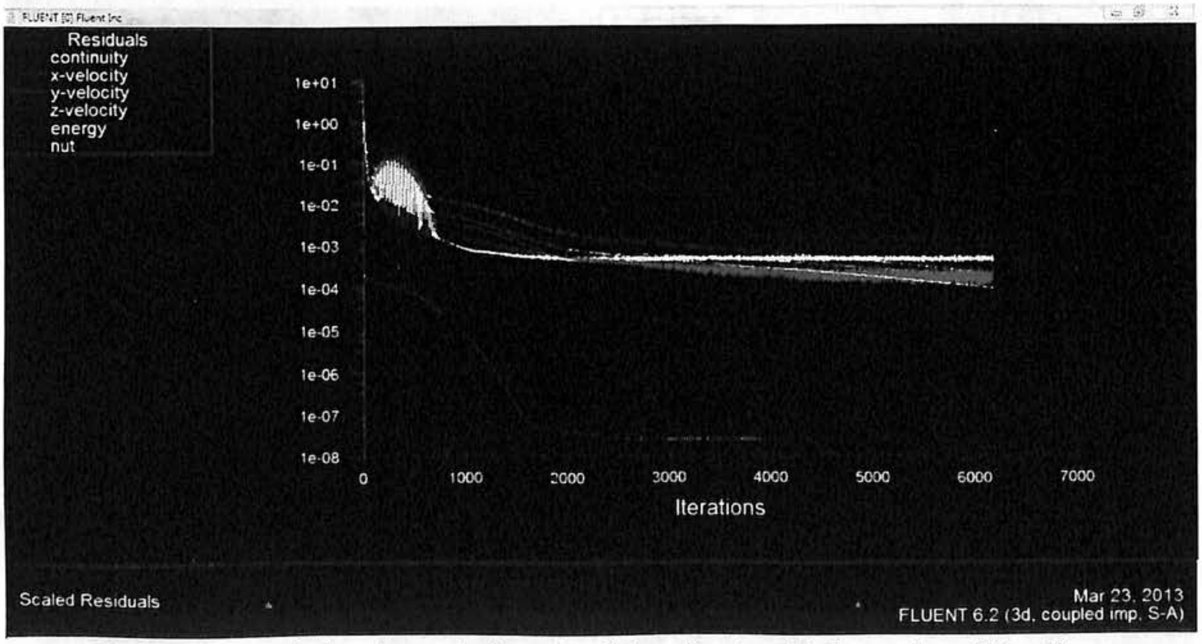

Figure 4.34: Graph showing variation of scaled residuals with the number of iterations

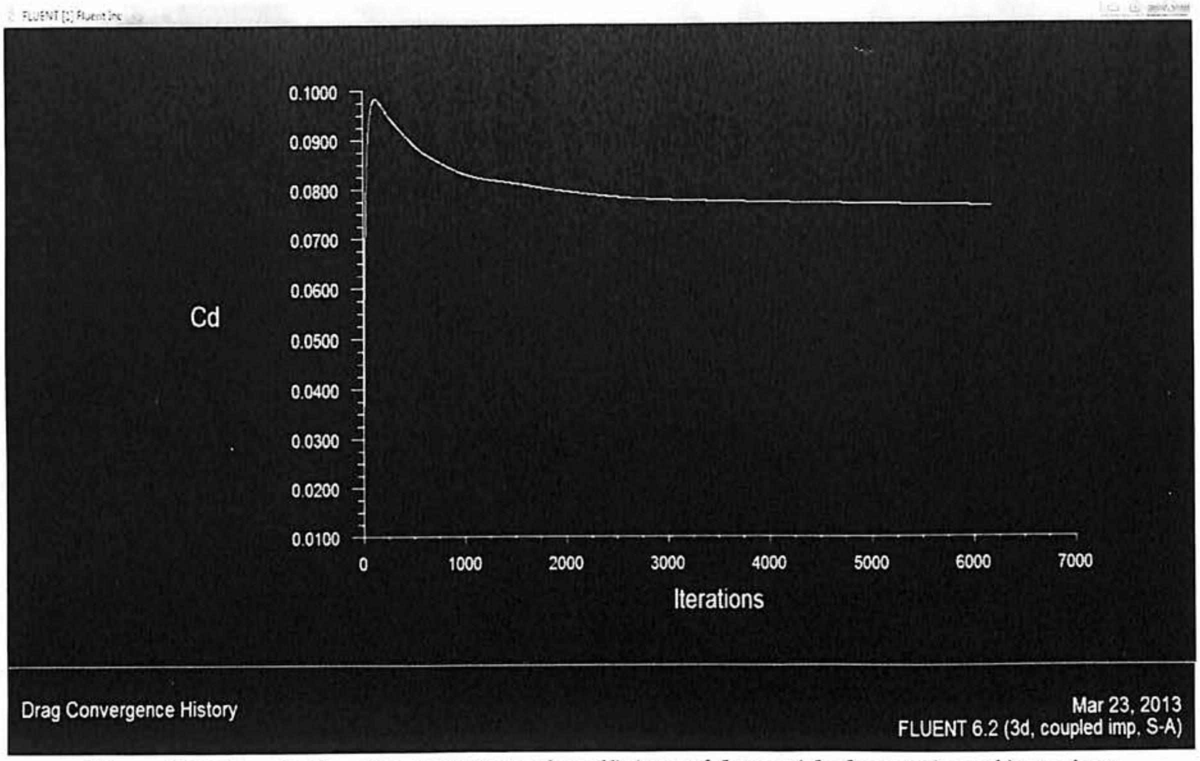

Figure 4.35: Graph showing variation of coefficient of drag with the number of iterations

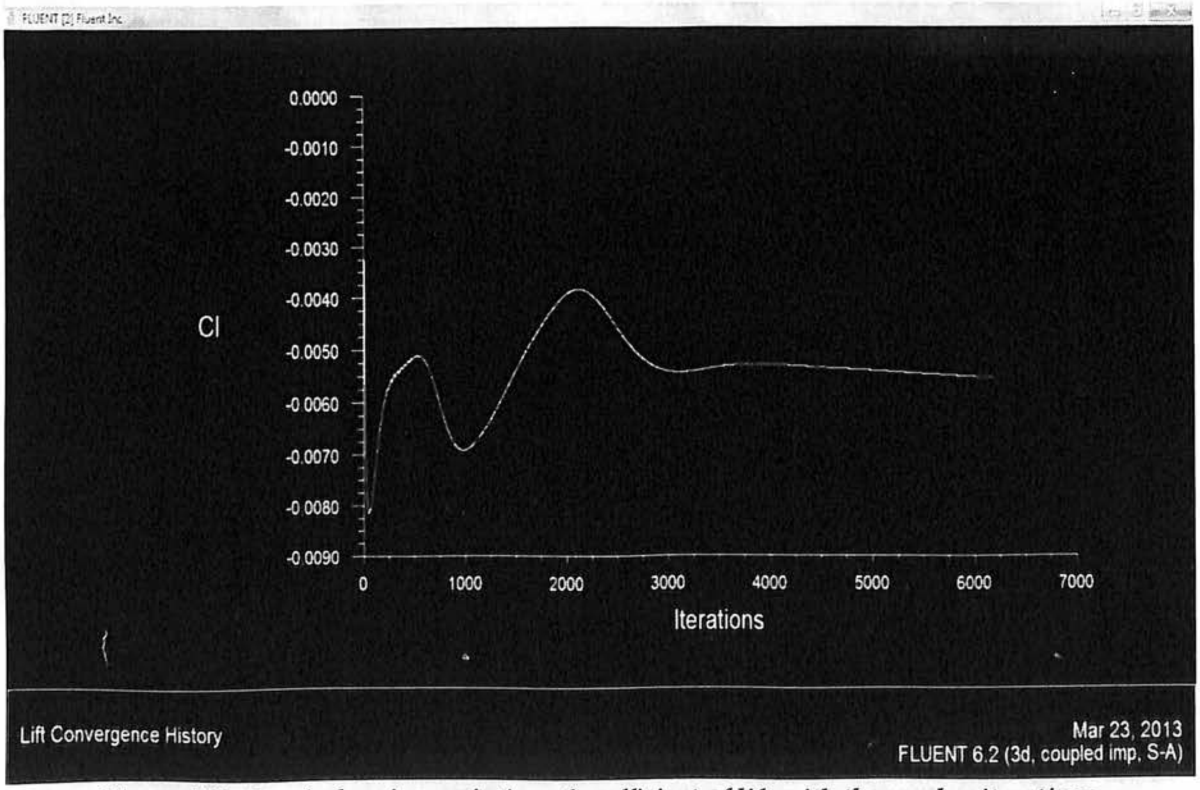

Figure 4.36: Graph showing variation of coefficient of lift with the number iterations

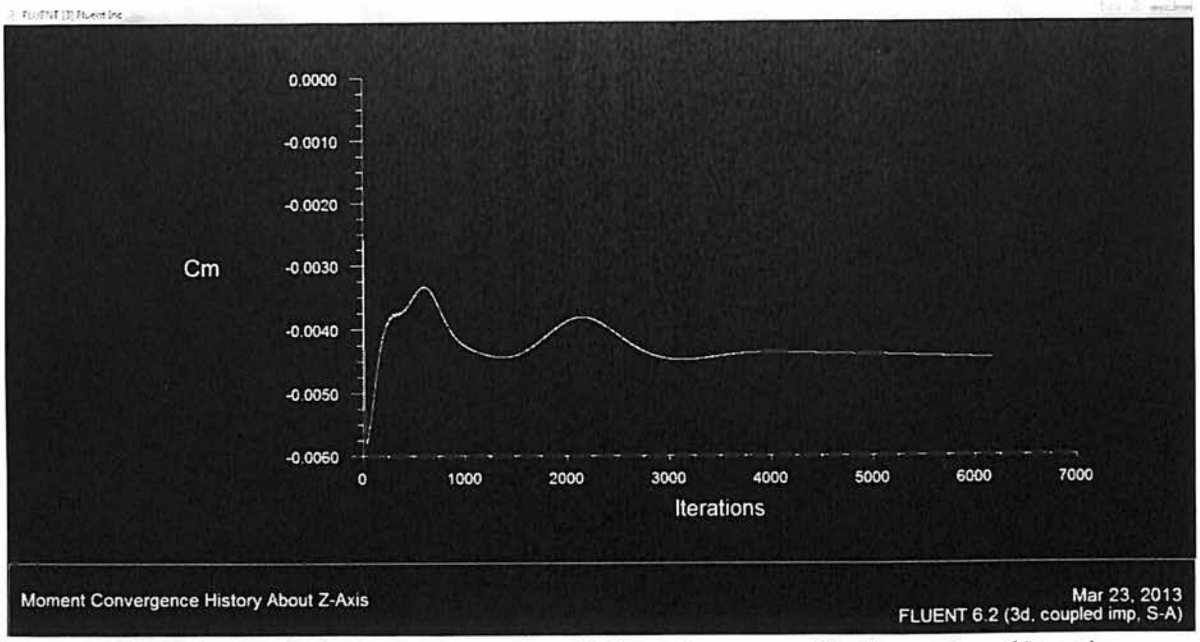

Figure 4.37: Graph showing variation of coefficient of moment with the number of iterations

4.4.2 RUDDER DEFLECTIONS

Following are the graphs are end product of simulations that were done on different deflections of rudder.

For rudder deflection of 10 degrees left:

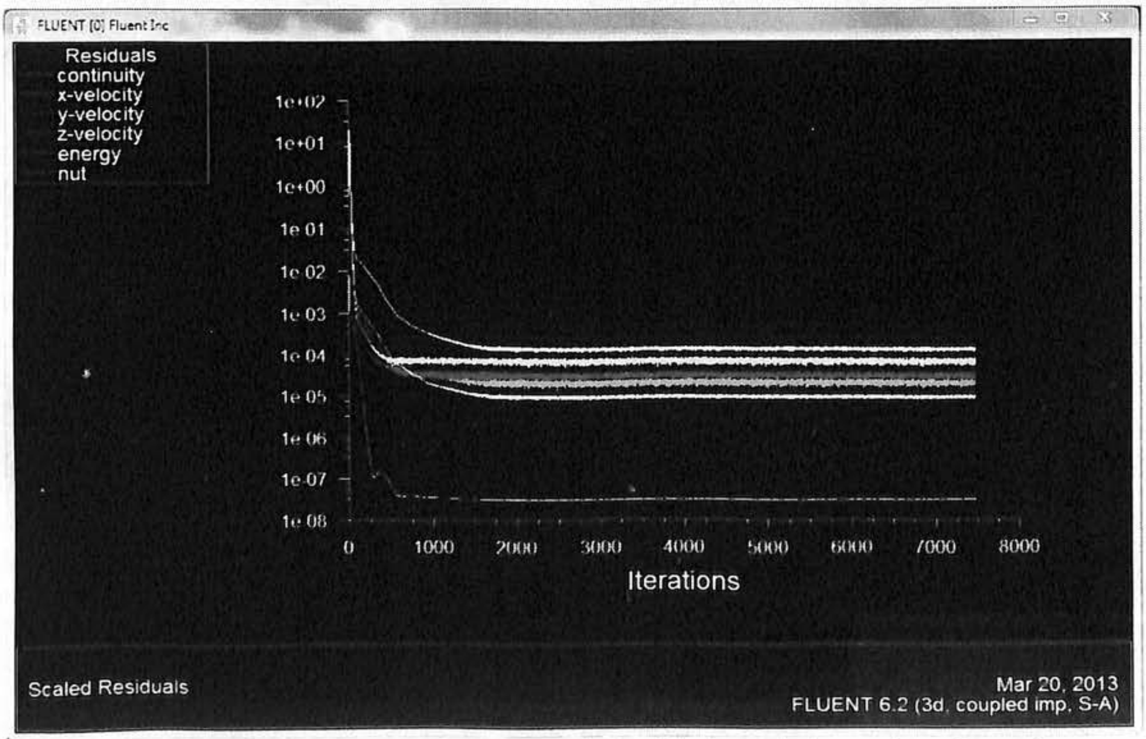

Figure 4.38: Graph showing variation of scaled residuals with the number of iterations

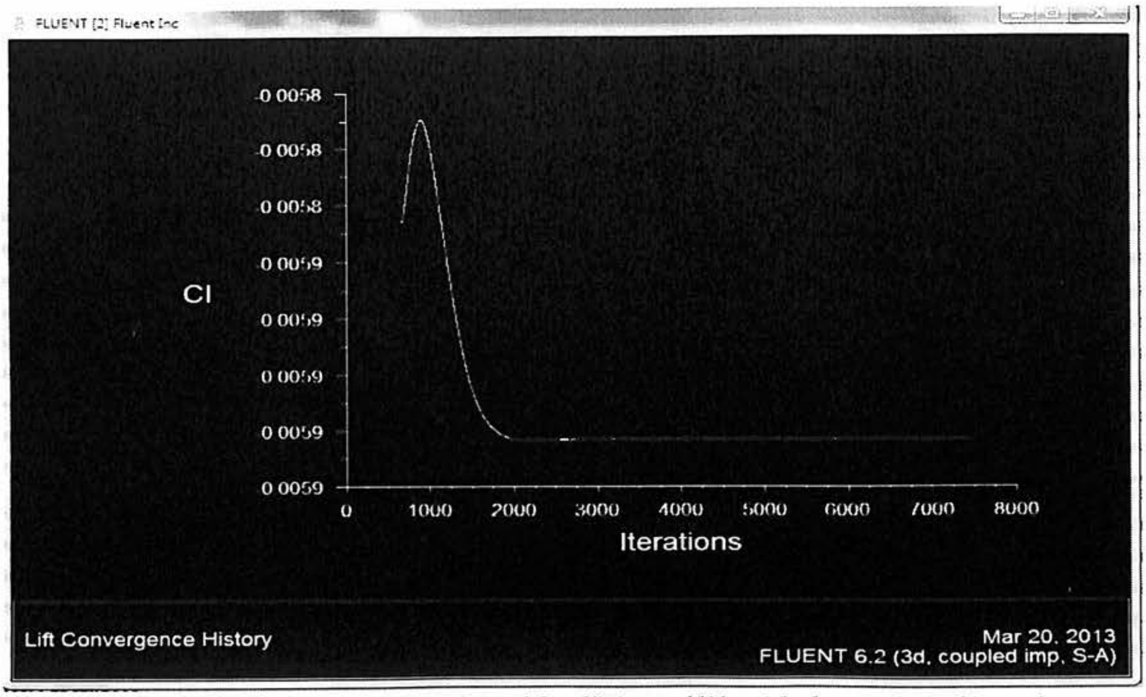

Figure 4.39: Graph showing variation of Coefficient of lift with the number of iterations

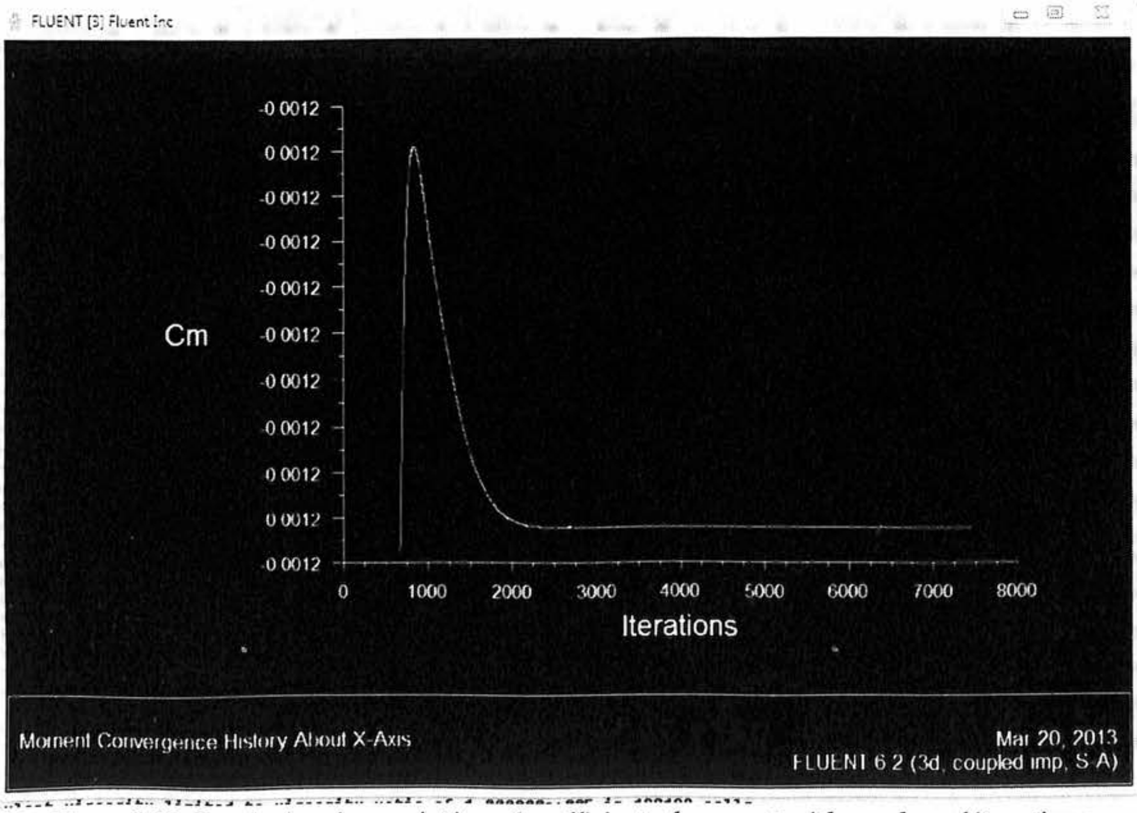

Figure 4.40: Graph showing variation of coefficient of moment with number of iterations

4.4.3 AILERRON DEFLECTION

Following graphs are the end result of the simulations done for Aileron deflection of negative 5 degrees:

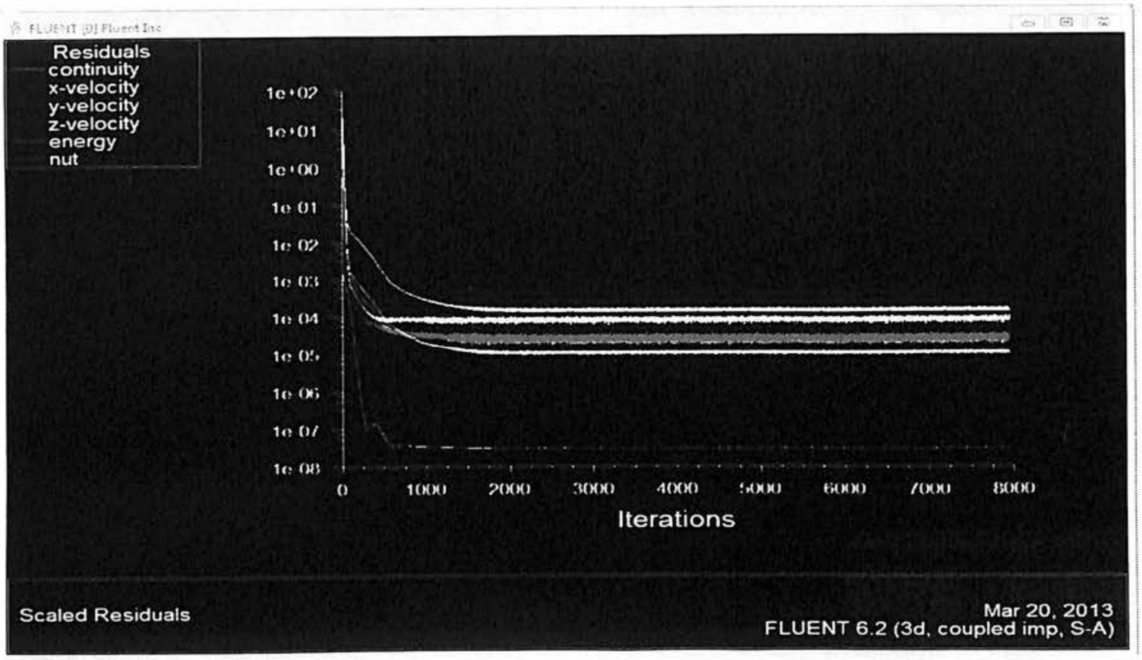

Figure 4.41: Graph showing variations of scaled residuals

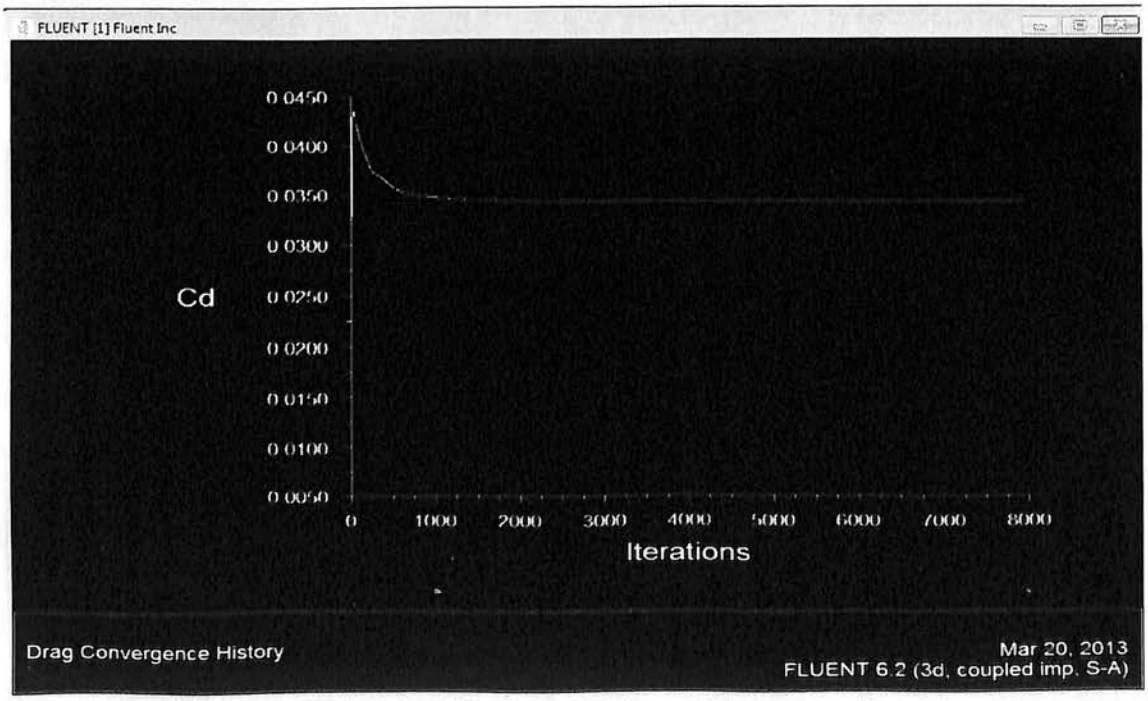

Figure 4.42: Variation of Coefficient of Drag with number of iterations

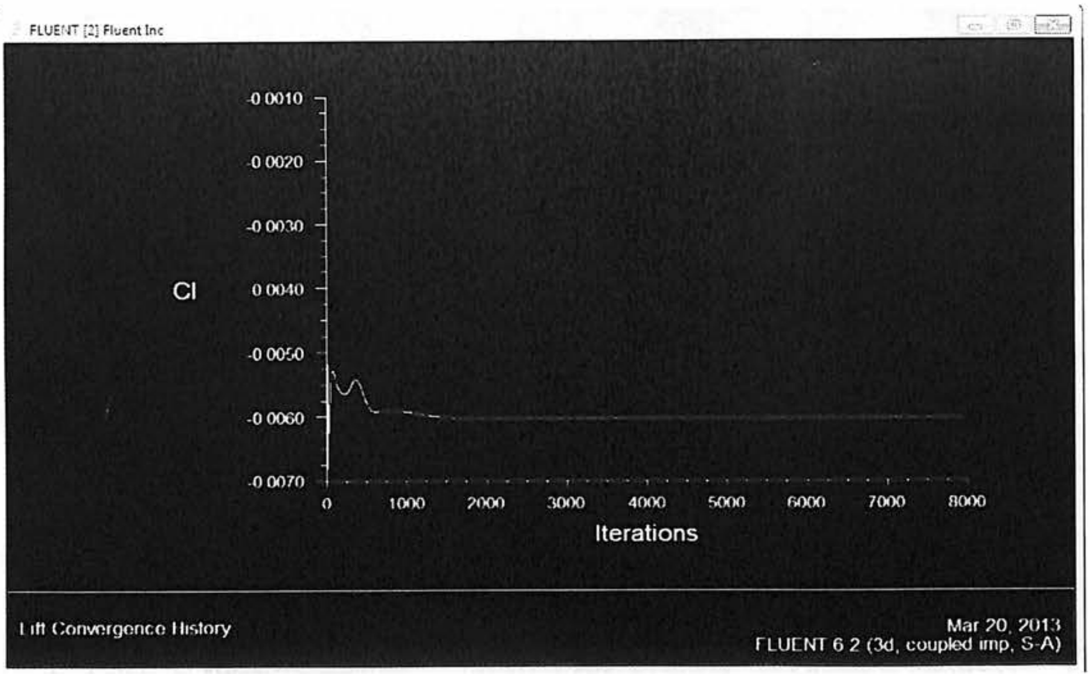

Figure 4.43: Graph showing variation of coefficient of lift with the number of iterations

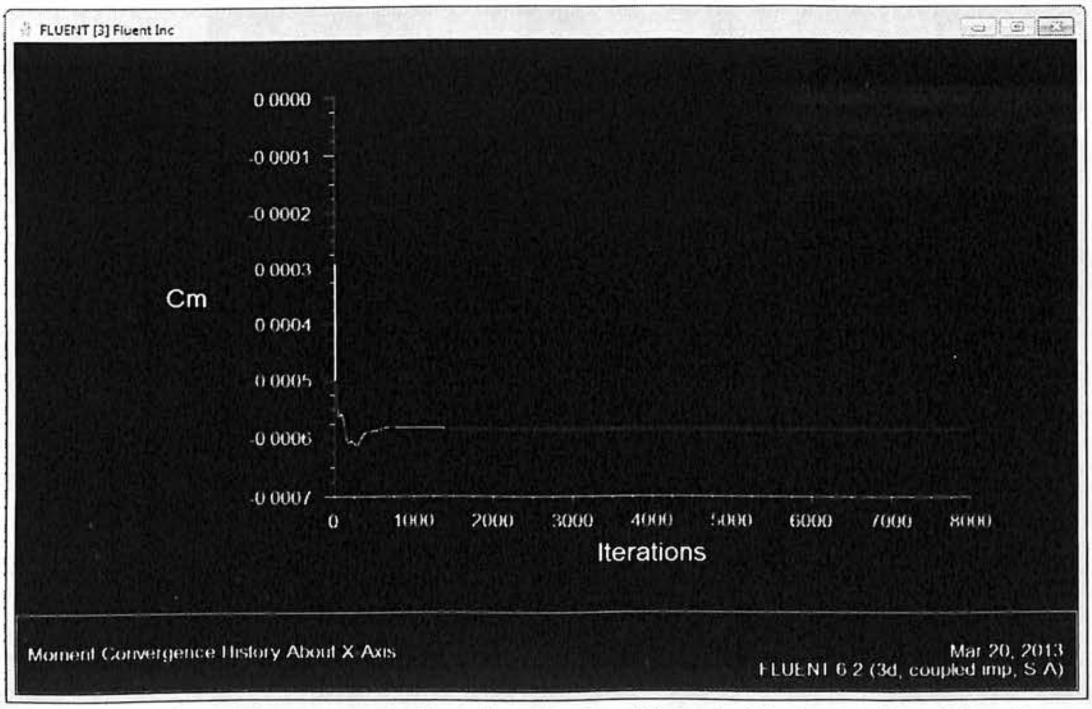

Figure 4.44: Graph showing variation of moment coefficient with the number of iterations

Similarly there are graphs showing results for aileron deflection of 5 degrees positive and 10 degrees negative.

Results of all the graphs derived above are discussed in the design of experiment ahead.

4.5 Design of experiment

Table 1: Design of experiment

S.no	α	δ_e	δ_r	δ_a	C_L	C_D	C_{mx}	C_{my}	C_{mz}
1	2	0	0	0	0.061447	0.074944	0.000068788	-0.00019897	0.013175
2	4	0	0	0	0.13503	0.07429	0.0000048117	-0.00017904	0.035199
3	6	0	0	0	0.20972	0.073255	0.00004025	-0.00020284	0.057513
4	0	-10	0	0	-0.0051742	0.076032	0.000040196	0 0000030019	-0.0044625
5	0	-5	0	0	-0.020329	0.076508	0.000086659	-0.00031675	-0.014237
6	0	5	0	0	0.0002192	0.077827	-0.000050019	-0.00027791	-0.00010322
7	0	0	-10	0	-0.014452	0.076567	0.0010396	-0.0062416	-0.010749
8	0	0	5	0	-0.013721	0.075828	-0.000458	0.0026832	-0.0098885
9	0	0	10	0	-0.013974	0.077514	-0.0010002	0.0058333	-0.010489
10	0	0	0	-10	-0.013095	0.078106	-0.002598	-0.0012136	-0.0094334
11	0	0	0	-5	-0.013329	0.076008	-0.0012893	-0.0007777	-0.0093993
12	0	0	0	5	-0.012402	0.076509	0.0011925	0.00045362	-0.0092854
13	0	0	0	10	-0.012574	0.077609	0.002641	0.00087029	-0.0093678

CHAPTER 5: RECONSTRUCTION OF AERODYNAMIC FUNCTION

5.1 Multiquardic Radial Basic Function: Interpolations

When we are using mathematical modelling, equations can be simplified as single-layer neural networks, also called "radial basis network functions". These can solve the functions for brief intervals with adequate accuracy via interpolation if a fairly large number of these functions are exercised.

In other words when we are aware of the values of a function at certain points, we can use radial basis network functions for interpolations to derive the values at as many points we need between the two known points. [8]

5.2 Generic Equations

The general formula for the generic equations of all the functions is:

$$f^1 = \sum_{i=1}^{13} \lambda_i r_i$$

Where, "r_i" is defined by:

$$r_i = \sqrt{(\alpha - \alpha_i)^2 + (\delta - \delta_e)^2 + (\delta - \delta_r)^2 + (\delta - \delta_a)^2 + 5^2}$$

For deriving values for a particular function our formula evolves to:

$$f^2{}_j = \sum_{i=1}^{13} \lambda_i r_{ij}$$

---

[11] *For equations in details refer to APPENDIX A-1*

[2] *For detailed equations refer to APPENDIX A-2*

Where "r_{ij}" is defined by:

$$r_{ij} = \sqrt{(\alpha_j - \alpha_i)^2 + (\delta_{ej} - \delta_{ei})^2 + (\delta_{rj} - \delta_{ri})^2 + (\delta_{aj} - \delta_{ai})^2 + 5^2}$$

5.3 Solutions of The Equations' Unknown Variables

For solution of the unknown variables present in the generic equations, a java code has been used in which these generic equations were presented in matrix form which was individually developed for each parameter. Then these matrices were solved using the Gauss Elimination Method[3].

The solutions that were worked outC_L are:

Table 2: λ values for coefficient of lift

Λ	Value
λ_1	-0.028552649060467065
λ_2	0.0806538809698669
λ_3	-0.0884871654002307
λ_4	0.015423706520055225
λ_5	-0.028263501814250403
λ_6	-0.0031962335194168044
λ_7	0.007341641909016804
λ_8	0.008929576191342284
λ_9	0.0023129706339276098
λ_{10}	-0.009396675511478311
λ_{11}	0.02883237943829598
λ_{12}	0.01695973772250218
λ_{13}	-0.0011075102277246205

[3]*For the Java code refer APPENDIX B-1*

The solutions that were worked out C_D are:

Table 3: λ values for coefficient of drag

Λ	Value
λ_1	0.007968017798372404
λ_2	-0.013204251707559591
λ_3	0.007535227310795274
λ_4	0.002574301851520408
λ_5	-0.003352030429656648
λ_6	7.516347928957228E-5
λ_7	0.001568978094130503
λ_8	-8.770111970958938E-4
λ_9	0.0019992138748182136
λ_{10}	0.002057763410466018
λ_{11}	-0.001890461551118472
λ_{12}	-0.002495317241962822
λ_{13}	0.002428140418539001

The solutions that were worked out C_{MX} are:

Table 4: λ values for coefficient of moment in x-direction

Λ	Value
λ_1	-8.429262469756175E-4
λ_2	0.0010913915570664318
λ_3	-4.744069546503994E-4
λ_4	1.7458584955775076E-5
λ_5	-7.400481988253531E-6
λ_6	3.9404650587482234E-5
λ_7	-6.872239720221685E-5
λ_8	-7.43455830106406E-5

λ_9	1.03516684331945E-4
λ_{10}	2.059783625260486E-4
λ_{11}	-3.650705593024977E-6
λ_{12}	3.4611550851941443E-4
λ_{13}	-3.2529209720538906E-4

The solutions that were worked out C_{MY} are:

Table 5: λ values for coefficient of moment in y-direction

Λ	Value
λ_1	0.0017613606286960544
λ_2	-0.003092205016823466
λ_3	0.0015332136779497632
λ_4	-9.320712865723847E-5
λ_5	8.985039840180477E-5
λ_6	-4.824883106742044E-5
λ_7	4.062964144407101E-4
λ_8	4.136837899216143E-4
λ_9	-5.985129507184427E-4
λ_{10}	1.209151426172088E-4
λ_{11}	-2.351484956718388E-4
λ_{12}	-2.855482843007517E-4
λ_{13}	1.5675045623587656E-5

The solutions that were worked out C_{MZ} are:

Table 6: λ values for coefficient of moment in z-direction

Λ	Value
λ_1	-0.018069250481488483
λ_2	0.030711858583905238
λ_3	-0.028157647441959973

λ_4	-0.003908730183439155
λ_5	0.008212312314538821
λ_6	3.9763943451074583E-4
λ_7	0.002031664906653137
λ_8	0.001685256164696786
λ_9	0.0010717435474446335
λ_{10}	6.444013281933587E-4
λ_{11}	0.0025441724660877677
λ_{12}	0.0024842579955488436
λ_{13}	6.737356978464467E-4

Table 7: Lookup Table[4]

S No.	α	δ_e	δ_r	δ_a	C_L	C_D	C_{mx}	C_{my}	C_{mz}
1	2	-10	-10	-10	0.0393	0.09436	-0.00093	-0.00468	0.005217
2	2	-10	-10	-9	0.037603	0.092577	-0.00077	-0.00472	0.005163
3	2	-10	-10	-8	0.035991	0.090956	-0.00061	-0.00475	0.005123
4	2	-10	-10	-7	0.034477	0.089502	-0.00043	-0.00477	0.005096
5	2	-10	-10	-6	0.033072	0.088221	-0.00026	-0.00478	0.005079
6	2	-10	-10	-5	0.031779	0.087119	-7.6E-05	-0.00479	0.00507
7	2	-10	-10	-4	0.030592	0.086202	0.000108	-0.00478	0.005067
8	2	-10	-10	-3	0.029505	0.085477	0.000294	-0.00476	0.005066
9	2	-10	-10	-2	0.028505	0.084949	0.00048	-0.00473	0.005068
10	2	-10	-10	-1	0.027577	0.084625	0.000666	-0.00468	0.00507
11	2	-10	-10	0	0.026707	0.084507	0.00085	-0.00461	0.005073
12	2	-10	-10	1	0.025883	0.084596	0.001032	-0.00453	0.005075
13	2	-10	-10	2	0.025096	0.08489	0.001211	-0.00443	0.005078
14	2	-10	-10	3	0.024344	0.085384	0.001387	-0.00432	0.005081
15	2	-10	-10	4	0.023627	0.086072	0.001558	-0.0042	0.005086
16	2	-10	-10	5	0.022951	0.086948	0.001726	-0.00407	0.005093
17	2	-10	-10	6	0.022328	0.088004	0.001888	-0.00394	0.005105
18	2	-10	-10	7	0.021769	0.089235	0.002046	-0.00379	0.005124
19	2	-10	-10	8	0.021288	0.090635	0.002197	-0.00365	0.005152
20	2	-10	-10	9	0.020896	0.092199	0.002342	-0.00351	0.005192
21	2	-10	-10	10	0.020601	0.093924	0.002479	-0.00337	0.005247
22	2	-10	-9	-10	0.040124	0.092642	-0.00103	-0.00437	0.005307
23	2	-10	-9	-9	0.03849	0.090855	-0.00087	-0.0044	0.005284
24	2	-10	-9	-8	0.036953	0.089233	-0.0007	-0.00442	0.005278
25	2	-10	-9	-7	0.035529	0.087781	-0.00053	-0.00443	0.005287
26	2	-10	-9	-6	0.034224	0.086501	-0.00035	-0.00444	0.005307
27	2	-10	-9	-5	0.033038	0.085401	-0.00016	-0.00444	0.005333
28	2	-10	-9	-4	0.03196	0.084487	2.51E-05	-0.00442	0.005363
29	2	-10	-9	-3	0.030973	0.083763	0.000215	-0.0044	0.005391
30	2	-10	-9	-2	0.030056	0.083237	0.000405	-0.00436	0.005415
31	2	-10	-9	-1	0.029186	0.082915	0.000595	-0.00431	0.005431
32	2	-10	-9	0	0.028344	0.082798	0.000784	-0.00424	0.005439
33	2	-10	-9	1	0.027511	0.082888	0.000971	-0.00415	0.005437
34	2	-10	-9	2	0.026679	0.083183	0.001154	-0.00406	0.005426
35	2	-10	-9	3	0.025847	0.083677	0.001335	-0.00395	0.005407
36	2	-10	-9	4	0.02502	0.084365	0.001511	-0.00383	0.005384
37	2	-10	-9	5	0.024213	0.085239	0.001684	-0.0037	0.005358
38	2	-10	-9	6	0.023443	0.086292	0.001852	-0.00356	0.005334
39	2	-10	-9	7	0.022732	0.08752	0.002014	-0.00343	0.005317
40	2	-10	-9	8	0.0221	0.088917	0.00217	-0.00329	0.005309
41	2	-10	-9	9	0.021565	0.09048	0.00232	-0.00315	0.005315

[4]*For Creation of lookup table refer python code from APPENDIX B-2*

42	2	-10	-9	10	0.021138	0.092205	0.002462	-0.00302	0.005339
43	2	-10	-8	-10	0.040972	0.091095	-0.00113	-0.00403	0.005404
44	2	-10	-8	-9	0.039403	0.089308	-0.00097	-0.00405	0.005413
45	2	-10	-8	-8	0.037947	0.087689	-0.0008	-0.00406	0.005444
46	2	-10	-8	-7	0.036618	0.086241	-0.00062	-0.00407	0.005491
47	2	-10	-8	-6	0.035422	0.084967	-0.00044	-0.00407	0.005551
48	2	-10	-8	-5	0.034353	0.083872	-0.00025	-0.00406	0.005617
49	2	-10	-8	-4	0.033393	0.082961	-6.1E-05	-0.00404	0.005684
50	2	-10	-8	-3	0.032518	0.082241	0.000132	-0.00401	0.005745
51	2	-10	-8	-2	0.031694	0.081717	0.000326	-0.00397	0.005794
52	2	-10	-8	-1	0.030891	0.081396	0.000521	-0.00391	0.005827
53	2	-10	-8	0	0.030081	0.081281	0.000713	-0.00384	0.00584
54	2	-10	-8	1	0.029242	0.081373	0.000904	-0.00375	0.005833
55	2	-10	-8	2	0.028364	0.081668	0.001092	-0.00365	0.005806
56	2	-10	-8	3	0.027447	0.082162	0.001278	-0.00354	0.005762
57	2	-10	-8	4	0.026504	0.082848	0.001459	-0.00343	0.005706
58	2	-10	-8	5	0.025555	0.083718	0.001637	-0.0033	0.005644
59	2	-10	-8	6	0.024629	0.084767	0.00181	-0.00317	0.005581
60	2	-10	-8	7	0.023757	0.085988	0.001978	-0.00304	0.005523
61	2	-10	-8	8	0.022965	0.087378	0.002139	-0.00291	0.005477
62	2	-10	-8	9	0.022278	0.088935	0.002294	-0.00277	0.005447
63	2	-10	-8	10	0.021714	0.090657	0.00244	-0.00265	0.005437
64	2	-10	-7	-10	0.041809	0.089722	-0.00123	-0.00368	0.005502
65	2	-10	-7	-9	0.040303	0.087939	-0.00107	-0.00369	0.005544
66	2	-10	-7	-8	0.038926	0.086326	-0.0009	-0.00369	0.005611
67	2	-10	-7	-7	0.037692	0.084885	-0.00072	-0.00369	0.005699
68	2	-10	-7	-6	0.036606	0.083618	-0.00054	-0.00368	0.005801
69	2	-10	-7	-5	0.035657	0.082529	-0.00035	-0.00366	0.005909
70	2	-10	-7	-4	0.034819	0.081623	-0.00015	-0.00363	0.006016
71	2	-10	-7	-3	0.034057	0.080906	4.62E-05	-0.0036	0.006113
72	2	-10	-7	-2	0.03333	0.080384	0.000244	-0.00355	0.006191
73	2	-10	-7	-1	0.032596	0.080065	0.000442	-0.00348	0.006243
74	2	-10	-7	0	0.031821	0.079951	0.000638	-0.00341	0.006263
75	2	-10	-7	1	0.030979	0.080045	0.000833	-0.00332	0.00625
76	2	-10	-7	2	0.030058	0.080342	0.001026	-0.00322	0.006205
77	2	-10	-7	3	0.02906	0.080836	0.001216	-0.00312	0.006133
78	2	-10	-7	4	0.028001	0.08152	0.001402	-0.003	0.006041
79	2	-10	-7	5	0.026911	0.082386	0.001585	-0.00288	0.005938
80	2	-10	-7	6	0.025827	0.083428	0.001763	-0.00276	0.005833
81	2	-10	-7	7	0.02479	0.08464	0.001936	-0.00263	0.005733
82	2	-10	-7	8	0.023838	0.08602	0.002103	-0.0025	0.005647

83	2	-10	-7	9	0.023	0.087568	0.002263	-0.00238	0.00558
84	2	-10	-7	10	0.022297	0.089283	0.002414	-0.00226	0.005537
85	2	-10	-6	-10	0.0426	0.088527	-0.00133	-0.00331	0.005594
86	2	-10	-6	-9	0.041151	0.086751	-0.00117	-0.00331	0.005669
87	2	-10	-6	-8	0.039845	0.085146	-0.001	-0.0033	0.005772
88	2	-10	-6	-7	0.0387	0.083714	-0.00082	-0.00329	0.0059
89	2	-10	-6	-6	0.037716	0.082455	-0.00063	-0.00327	0.006044
90	2	-10	-6	-5	0.036879	0.081373	-0.00044	-0.00324	0.006196
91	2	-10	-6	-4	0.036155	0.08047	-0.00024	-0.0032	0.006346
92	2	-10	-6	-3	0.035499	0.079754	-4.2E-05	-0.00316	0.006482
93	2	-10	-6	-2	0.03486	0.079233	0.000159	-0.0031	0.006592
94	2	-10	-6	-1	0.034191	0.078913	0.00036	-0.00304	0.006664
95	2	-10	-6	0	0.033452	0.0788	0.00056	-0.00296	0.006692
96	2	-10	-6	1	0.032613	0.078896	0.000759	-0.00287	0.006671
97	2	-10	-6	2	0.031659	0.079197	0.000955	-0.00277	0.006606
98	2	-10	-6	3	0.03059	0.079694	0.001149	-0.00267	0.006503
99	2	-10	-6	4	0.029427	0.080378	0.001341	-0.00256	0.006373
100	2	-10	-6	5	0.028205	0.081241	0.001528	-0.00244	0.006227
101	2	-10	-6	6	0.026972	0.082275	0.001712	-0.00232	0.006078
102	2	-10	-6	7	0.025779	0.083477	0.00189	-0.0022	0.005936
103	2	-10	-6	8	0.024673	0.084846	0.002062	-0.00209	0.005809
104	2	-10	-6	9	0.02369	0.086383	0.002227	-0.00197	0.005706
105	2	-10	-6	10	0.022858	0.088089	0.002383	-0.00186	0.005631
106	2	-10	-5	-10	0.043313	0.087515	-0.00144	-0.00293	0.005677
107	2	-10	-5	-9	0.04191	0.085746	-0.00127	-0.00292	0.005781
108	2	-10	-5	-8	0.040663	0.084151	-0.0011	-0.0029	0.005918
109	2	-10	-5	-7	0.039591	0.082728	-0.00092	-0.00287	0.006083
110	2	-10	-5	-6	0.038695	0.081478	-0.00073	-0.00284	0.006269
111	2	-10	-5	-5	0.037952	0.0804	-0.00053	-0.0028	0.006465
112	2	-10	-5	-4	0.037322	0.079499	-0.00033	-0.00276	0.006659
113	2	-10	-5	-3	0.036751	0.078781	-0.00013	-0.0027	0.006836
114	2	-10	-5	-2	0.036183	0.078256	7.16E-05	-0.00264	0.006979
115	2	-10	-5	-1	0.035565	0.077933	0.000275	-0.00257	0.007075
116	2	-10	-5	0	0.034858	0.077821	0.000479	-0.00249	0.00711
117	2	-10	-5	1	0.034028	0.07792	0.000681	-0.0024	0.007082
118	2	-10	-5	2	0.033057	0.078226	0.000881	-0.0023	0.006995
119	2	-10	-5	3	0.03194	0.078729	0.001079	-0.0022	0.006858
120	2	-10	-5	4	0.030696	0.079417	0.001275	-0.00209	0.006687
121	2	-10	-5	5	0.029364	0.080279	0.001467	-0.00198	0.006498
122	2	-10	-5	6	0.028002	0.081307	0.001656	-0.00187	0.006305
123	2	-10	-5	7	0.02667	0.0825	0.001839	-0.00176	0.006122

The lookup table generated contains the calculated reactions for the individual inputs. The purpose of the project was to discover these reactions for the aeroplane (MiG-21) so that we could know beforehand the inflight behaviour of the aircraft. This can serve two purposes, first one for reference in the simulator.

Whenever a human pilot would give an input in the simulator, the simulator software would refer the look-up tables provided and draft a reaction of the simulated aircraft. This way for every input a corresponding value of forces will be generated by the software and the pilots would then be able to know the outcome of their inputs for the real aircraft well below flying one.

A second purpose these lookup tables could deliver is for the autopilots. Just like the simulators but in the reverse order, these lookup tables will be referred by an in-flight active autopilot to device a proper counter reaction in control surfaces for environmental disturbances that are generated during the flight. The crosswinds that the aeroplane will encounter will exert moments about the aircraft, these moments that will try putting the aircraft out of its trajectory and the autopilot will sense how much has the aeroplane travelled out of its path and then device a reaction needed to counter that sequence using the combinations of control surface movements in the lookup tables.

CHAPTER 7: CONCLUSION

This project has focused on the study of the different levels of deflection of various control surfaces and their corresponding effects on the aircraft, so the user could gain familiarity with the action-reaction balance of stick movements and the aircraft manoeuvres.

This is achieved by calculating one by one the effects of every control surface at various degrees of deflection. The moments were noted for a set of fixed interval deflections of the control surfaces. Then the values were used to generate a transform function for the relationship between the degree of deflection and corresponding moments generated. This was done at zero degree, five degrees and ten degrees, both negative and positive, for rudders, ailerons and elevators. This discreet data was transformed into continues data with multiquardic radial basis function for each control surface. And finally this was all compiled to generate a lookup table that defines reactions and aircraft movements for their corresponding pilot inputs and atmospheric disturbances.

There were some issues that were encountered during the project that had to be dealt on the cost of precision. Most of these were encountered during the designing of the aeroplane models, the chief one was, in order to keep the control surfaces from overlapping into the model volume some margin was introduced which in the actual aircraft would have been some mechanism that on this scale simply could not be designed into our model.

The results that we have obtained using mathematical modelling are adequate for the solution of individual inputs or simple combinations of these individual inputs and the lookup table generated contains the calculated reactions for the individual inputs which are fairly precise at predicting their corresponding outputs. In stark contrast, the purpose of the project to discover the stick inputs and their corresponding manoeuvring effects on theaeroplane in focus (MiG-21) is acceptably achieved.

APPENDIX A-1

$$
\begin{aligned}
&C_D(\alpha, \delta_e, \delta_r, \delta_a) \\
&= \lambda_1\sqrt{(\alpha-\alpha_1)^2+(\delta_e-\delta_{e1})^2+(\delta_r-\delta_{r1})^2+(\delta_a-\delta_{a1})^2+5^2} \\
&+ \lambda_2\sqrt{(\alpha-\alpha_2)^2+(\delta_e-\delta_{e2})^2+(\delta_r-\delta_{r2})^2+(\delta_a-\delta_{a2})^2+5^2} \\
&+ \lambda_3\sqrt{(\alpha-\alpha_3)^2+(\delta_e-\delta_{e3})^2+(\delta_r-\delta_{r3})^2+(\delta_a-\delta_{a3})^2+5^2} \\
&+ \lambda_4\sqrt{(\alpha-\alpha_4)^2+(\delta_e-\delta_{e4})^2+(\delta_r-\delta_{r4})^2+(\delta_a-\delta_{a4})^2+5^2} \\
&+ \lambda_5\sqrt{(\alpha-\alpha_5)^2+(\delta_e-\delta_{e5})^2+(\delta_r-\delta_{r5})^2+(\delta_a-\delta_{a5})^2+5^2} \\
&+ \lambda_6\sqrt{(\alpha-\alpha_6)^2+(\delta_e-\delta_{e6})^2+(\delta_r-\delta_{r6})^2+(\delta_a-\delta_{a6})^2+5^2} \\
&+ \lambda_7\sqrt{(\alpha-\alpha_7)^2+(\delta_e-\delta_{e7})^2+(\delta_r-\delta_{r7})^2+(\delta_a-\delta_{a7})^2+5^2} \\
&+ \lambda_8\sqrt{(\alpha-\alpha_8)^2+(\delta_e-\delta_{e8})^2+(\delta_r-\delta_{r8})^2+(\delta_a-\delta_{a8})^2+5^2} \\
&+ \lambda_9\sqrt{(\alpha-\alpha_9)^2+(\delta_e-\delta_{e9})^2+(\delta_r-\delta_{r9})^2+(\delta_a-\delta_{a9})^2+5^2} \\
&+ \lambda_{10}\sqrt{(\alpha-\alpha_{10})^2+(\delta_e-\delta_{e10})^2+(\delta_r-\delta_{r10})^2+(\delta_a-\delta_{a10})^2+5^2} \\
&+ \lambda_{11}\sqrt{(\alpha-\alpha_{11})^2+(\delta_e-\delta_{e11})^2+(\delta_r-\delta_{r11})^2+(\delta_a-\delta_{a11})^2+5^2} \\
&+ \lambda_{12}\sqrt{(\alpha-\alpha_{12})^2+(\delta_e-\delta_{e12})^2+(\delta_r-\delta_{r12})^2+(\delta_a-\delta_{a12})^2+5^2} \\
&+ \lambda_{13}\sqrt{(\alpha-\alpha_{13})^2+(\delta_e-\delta_{e13})^2+(\delta_r-\delta_{r13})^2+(\delta_a-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$
\begin{aligned}
&C_L(\alpha,\delta_e,\delta_r,\delta_a)\\
&= \lambda_1\sqrt{(\alpha-\alpha_1)^2+(\delta_e-\delta_{e1})^2+(\delta_r-\delta_{r1})^2+(\delta_a-\delta_{a1})^2+5^2}\\
&+ \lambda_2\sqrt{(\alpha-\alpha_2)^2+(\delta_e-\delta_{e2})^2+(\delta_r-\delta_{r2})^2+(\delta_a-\delta_{a2})^2+5^2}\\
&+ \lambda_3\sqrt{(\alpha-\alpha_3)^2+(\delta_e-\delta_{e3})^2+(\delta_r-\delta_{r3})^2+(\delta_a-\delta_{a3})^2+5^2}\\
&+ \lambda_4\sqrt{(\alpha-\alpha_4)^2+(\delta_e-\delta_{e4})^2+(\delta_r-\delta_{r4})^2+(\delta_a-\delta_{a4})^2+5^2}\\
&+ \lambda_5\sqrt{(\alpha-\alpha_5)^2+(\delta_e-\delta_{e5})^2+(\delta_r-\delta_{r5})^2+(\delta_a-\delta_{a5})^2+5^2}\\
&+ \lambda_6\sqrt{(\alpha-\alpha_6)^2+(\delta_e-\delta_{e6})^2+(\delta_r-\delta_{r6})^2+(\delta_a-\delta_{a6})^2+5^2}\\
&+ \lambda_7\sqrt{(\alpha-\alpha_7)^2+(\delta_e-\delta_{e7})^2+(\delta_r-\delta_{r7})^2+(\delta_a-\delta_{a7})^2+5^2}\\
&+ \lambda_8\sqrt{(\alpha-\alpha_8)^2+(\delta_e-\delta_{e8})^2+(\delta_r-\delta_{r8})^2+(\delta_a-\delta_{a8})^2+5^2}\\
&+ \lambda_9\sqrt{(\alpha-\alpha_9)^2+(\delta_e-\delta_{e9})^2+(\delta_r-\delta_{r9})^2+(\delta_a-\delta_{a9})^2+5^2}\\
&+ \lambda_{10}\sqrt{(\alpha-\alpha_{10})^2+(\delta_e-\delta_{e10})^2+(\delta_r-\delta_{r10})^2+(\delta_a-\delta_{a10})^2+5^2}\\
&+ \lambda_{11}\sqrt{(\alpha-\alpha_{11})^2+(\delta_e-\delta_{e11})^2+(\delta_r-\delta_{r11})^2+(\delta_a-\delta_{a11})^2+5^2}\\
&+ \lambda_{12}\sqrt{(\alpha-\alpha_{12})^2+(\delta_e-\delta_{e12})^2+(\delta_r-\delta_{r12})^2+(\delta_a-\delta_{a12})^2+5^2}\\
&+ \lambda_{13}\sqrt{(\alpha-\alpha_{13})^2+(\delta_e-\delta_{e13})^2+(\delta_r-\delta_{r13})^2+(\delta_a-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$
\begin{aligned}
C_{mx}(\alpha, \delta_e, \delta_r, \delta_a)
&= \lambda_1\sqrt{(\alpha-\alpha_1)^2+(\delta_e-\delta_{e1})^2+(\delta_r-\delta_{r1})^2+(\delta_a-\delta_{a1})^2+5^2} \\
&+ \lambda_2\sqrt{(\alpha-\alpha_2)^2+(\delta_e-\delta_{e2})^2+(\delta_r-\delta_{r2})^2+(\delta_a-\delta_{a2})^2+5^2} \\
&+ \lambda_3\sqrt{(\alpha-\alpha_3)^2+(\delta_e-\delta_{e3})^2+(\delta_r-\delta_{r3})^2+(\delta_a-\delta_{a3})^2+5^2} \\
&+ \lambda_4\sqrt{(\alpha-\alpha_4)^2+(\delta_e-\delta_{e4})^2+(\delta_r-\delta_{r4})^2+(\delta_a-\delta_{a4})^2+5^2} \\
&+ \lambda_5\sqrt{(\alpha-\alpha_5)^2+(\delta_e-\delta_{e5})^2+(\delta_r-\delta_{r5})^2+(\delta_a-\delta_{a5})^2+5^2} \\
&+ \lambda_6\sqrt{(\alpha-\alpha_6)^2+(\delta_e-\delta_{e6})^2+(\delta_r-\delta_{r6})^2+(\delta_a-\delta_{a6})^2+5^2} \\
&+ \lambda_7\sqrt{(\alpha-\alpha_7)^2+(\delta_e-\delta_{e7})^2+(\delta_r-\delta_{r7})^2+(\delta_a-\delta_{a7})^2+5^2} \\
&+ \lambda_8\sqrt{(\alpha-\alpha_8)^2+(\delta_e-\delta_{e8})^2+(\delta_r-\delta_{r8})^2+(\delta_a-\delta_{a8})^2+5^2} \\
&+ \lambda_9\sqrt{(\alpha-\alpha_9)^2+(\delta_e-\delta_{e9})^2+(\delta_r-\delta_{r9})^2+(\delta_a-\delta_{a9})^2+5^2} \\
&+ \lambda_{10}\sqrt{(\alpha-\alpha_{10})^2+(\delta_e-\delta_{e10})^2+(\delta_r-\delta_{r10})^2+(\delta_a-\delta_{a10})^2+5^2} \\
&+ \lambda_{11}\sqrt{(\alpha-\alpha_{11})^2+(\delta_e-\delta_{e11})^2+(\delta_r-\delta_{r11})^2+(\delta_a-\delta_{a11})^2+5^2} \\
&+ \lambda_{12}\sqrt{(\alpha-\alpha_{12})^2+(\delta_e-\delta_{e12})^2+(\delta_r-\delta_{r12})^2+(\delta_a-\delta_{a12})^2+5^2} \\
&+ \lambda_{13}\sqrt{(\alpha-\alpha_{13})^2+(\delta_e-\delta_{e13})^2+(\delta_r-\delta_{r13})^2+(\delta_a-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$
\begin{aligned}
& C_{my}(\alpha, \delta_e, \delta_r, \delta_a) \\
& = \lambda_1\sqrt{(\alpha-\alpha_1)^2+(\delta_e-\delta_{e1})^2+(\delta_r-\delta_{r1})^2+(\delta_a-\delta_{a1})^2+5^2} \\
& + \lambda_2\sqrt{(\alpha-\alpha_2)^2+(\delta_e-\delta_{e2})^2+(\delta_r-\delta_{r2})^2+(\delta_a-\delta_{a2})^2+5^2} \\
& + \lambda_3\sqrt{(\alpha-\alpha_3)^2+(\delta_e-\delta_{e3})^2+(\delta_r-\delta_{r3})^2+(\delta_a-\delta_{a3})^2+5^2} \\
& + \lambda_4\sqrt{(\alpha-\alpha_4)^2+(\delta_e-\delta_{e4})^2+(\delta_r-\delta_{r4})^2+(\delta_a-\delta_{a4})^2+5^2} \\
& + \lambda_5\sqrt{(\alpha-\alpha_5)^2+(\delta_e-\delta_{e5})^2+(\delta_r-\delta_{r5})^2+(\delta_a-\delta_{a5})^2+5^2} \\
& + \lambda_6\sqrt{(\alpha-\alpha_6)^2+(\delta_e-\delta_{e6})^2+(\delta_r-\delta_{r6})^2+(\delta_a-\delta_{a6})^2+5^2} \\
& + \lambda_7\sqrt{(\alpha-\alpha_7)^2+(\delta_e-\delta_{e7})^2+(\delta_r-\delta_{r7})^2+(\delta_a-\delta_{a7})^2+5^2} \\
& + \lambda_8\sqrt{(\alpha-\alpha_8)^2+(\delta_e-\delta_{e8})^2+(\delta_r-\delta_{r8})^2+(\delta_a-\delta_{a8})^2+5^2} \\
& + \lambda_9\sqrt{(\alpha-\alpha_9)^2+(\delta_e-\delta_{e9})^2+(\delta_r-\delta_{r9})^2+(\delta_a-\delta_{a9})^2+5^2} \\
& + \lambda_{10}\sqrt{(\alpha-\alpha_{10})^2+(\delta_e-\delta_{e10})^2+(\delta_r-\delta_{r10})^2+(\delta_a-\delta_{a10})^2+5^2} \\
& + \lambda_{11}\sqrt{(\alpha-\alpha_{11})^2+(\delta_e-\delta_{e11})^2+(\delta_r-\delta_{r11})^2+(\delta_a-\delta_{a11})^2+5^2} \\
& + \lambda_{12}\sqrt{(\alpha-\alpha_{12})^2+(\delta_e-\delta_{e12})^2+(\delta_r-\delta_{r12})^2+(\delta_a-\delta_{a12})^2+5^2} \\
& + \lambda_{13}\sqrt{(\alpha-\alpha_{13})^2+(\delta_e-\delta_{e13})^2+(\delta_r-\delta_{r13})^2+(\delta_a-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$
\begin{aligned}
&C_{mz}(\alpha, \delta_e, \delta_r, \delta_a) \\
&= \lambda_1\sqrt{(\alpha-\alpha_1)^2+(\delta_e-\delta_{e1})^2+(\delta_r-\delta_{r1})^2+(\delta_a-\delta_{a1})^2+5^2} \\
&+ \lambda_2\sqrt{(\alpha-\alpha_2)^2+(\delta_e-\delta_{e2})^2+(\delta_r-\delta_{r2})^2+(\delta_a-\delta_{a2})^2+5^2} \\
&+ \lambda_3\sqrt{(\alpha-\alpha_3)^2+(\delta_e-\delta_{e3})^2+(\delta_r-\delta_{r3})^2+(\delta_a-\delta_{a3})^2+5^2} \\
&+ \lambda_4\sqrt{(\alpha-\alpha_4)^2+(\delta_e-\delta_{e4})^2+(\delta_r-\delta_{r4})^2+(\delta_a-\delta_{a4})^2+5^2} \\
&+ \lambda_5\sqrt{(\alpha-\alpha_5)^2+(\delta_e-\delta_{e5})^2+(\delta_r-\delta_{r5})^2+(\delta_a-\delta_{a5})^2+5^2} \\
&+ \lambda_6\sqrt{(\alpha-\alpha_6)^2+(\delta_e-\delta_{e6})^2+(\delta_r-\delta_{r6})^2+(\delta_a-\delta_{a6})^2+5^2} \\
&+ \lambda_7\sqrt{(\alpha-\alpha_7)^2+(\delta_e-\delta_{e7})^2+(\delta_r-\delta_{r7})^2+(\delta_a-\delta_{a7})^2+5^2} \\
&+ \lambda_8\sqrt{(\alpha-\alpha_8)^2+(\delta_e-\delta_{e8})^2+(\delta_r-\delta_{r8})^2+(\delta_a-\delta_{a8})^2+5^2} \\
&+ \lambda_9\sqrt{(\alpha-\alpha_9)^2+(\delta_e-\delta_{e9})^2+(\delta_r-\delta_{r9})^2+(\delta_a-\delta_{a9})^2+5^2} \\
&+ \lambda_{10}\sqrt{(\alpha-\alpha_{10})^2+(\delta_e-\delta_{e10})^2+(\delta_r-\delta_{r10})^2+(\delta_a-\delta_{a10})^2+5^2} \\
&+ \lambda_{11}\sqrt{(\alpha-\alpha_{11})^2+(\delta_e-\delta_{e11})^2+(\delta_r-\delta_{r11})^2+(\delta_a-\delta_{a11})^2+5^2} \\
&+ \lambda_{12}\sqrt{(\alpha-\alpha_{12})^2+(\delta_e-\delta_{e12})^2+(\delta_r-\delta_{r12})^2+(\delta_a-\delta_{a12})^2+5^2} \\
&+ \lambda_{13}\sqrt{(\alpha-\alpha_{13})^2+(\delta_e-\delta_{e13})^2+(\delta_r-\delta_{r13})^2+(\delta_a-\delta_{a13})^2+5^2}
\end{aligned}
$$

APPENDIX A-2

$$
\begin{aligned}
&C_{L1}(\alpha_1, \delta_{e_1}, \delta_{r1}, \delta_{a1}) \\
&\quad = \lambda_1\sqrt{(\alpha_1-\alpha_1)^2+\left(\delta_{e_1}-\delta_{e_1}\right)^2+(\delta_{r1}-\delta r_1)^2+(\delta_{a1}-\delta_{a1})^2+5^2} \\
&\quad + \lambda_2\sqrt{(\alpha_1-\alpha_2)^2+\left(\delta_{e_1}-\delta_{e2}\right)^2+(\delta_{r1}-\delta_{r2})^2+(\delta_{a1}-\delta_{a2})^2+5^2} \\
&\quad + \lambda_3\sqrt{(\alpha_1-\alpha_3)^2+\left(\delta_{e_1}-\delta_{e3}\right)^2+(\delta_{r1}-\delta_{r3})^2+(\delta_{a1}-\delta_{a3})^2+5^2} \\
&\quad + \lambda_4\sqrt{(\alpha_1-\alpha_4)^2+\left(\delta_{e_1}-\delta_{e4}\right)^2+(\delta_{r1}-\delta_{r4})^2+(\delta_{a1}-\delta_{a4})^2+5^2} \\
&\quad + \lambda_5\sqrt{(\alpha_1-\alpha_5)^2+\left(\delta_{e_1}-\delta_{e5}\right)^2+(\delta_{r1}-\delta_{r5})^2+(\delta_{a1}-\delta_{a5})^2+5^2} \\
&\quad + \lambda_6\sqrt{(\alpha_1-\alpha_6)^2+\left(\delta_{e_1}-\delta_{e6}\right)^2+(\delta_{r1}-\delta_{r6})^2+(\delta_{a1}-\delta_{a6})^2+5^2} \\
&\quad + \lambda_7\sqrt{(\alpha_1-\alpha_7)^2+\left(\delta_{e_1}-\delta_{e7}\right)^2+(\delta_{r1}-\delta_{r7})^2+(\delta_{a1}-\delta_{a7})^2+5^2} \\
&\quad + \lambda_8\sqrt{(\alpha_1-\alpha_8)^2+\left(\delta_{e_1}-\delta_{e8}\right)^2+(\delta_{r1}-\delta_{r8})^2+(\delta_{a1}-\delta_{a8})^2+5^2} \\
&\quad + \lambda_9\sqrt{(\alpha_1-\alpha_9)^2+\left(\delta_{e_1}-\delta_{e9}\right)^2+(\delta_{r1}-\delta_{r9})^2+(\delta_{a1}-\delta_{a9})^2+5^2} \\
&\quad + \lambda_{10}\sqrt{(\alpha_1-\alpha_{10})^2+\left(\delta_{e_1}-\delta_{e10}\right)^2+(\delta_{r1}-\delta_{r10})^2+(\delta_{a1}-\delta_{a10})^2+5^2} \\
&\quad + \lambda_{11}\sqrt{(\alpha_1-\alpha_{11})^2+\left(\delta_{e_1}-\delta_{e11}\right)^2+(\delta_{r1}-\delta_{r11})^2+(\delta_{a1}-\delta_{a11})^2+5^2} \\
&\quad + \lambda_{12}\sqrt{(\alpha_1-\alpha_{12})^2+\left(\delta_{e_1}-\delta_{e12}\right)^2+(\delta_{r1}-\delta_{r12})^2+(\delta_{a1}-\delta_{a12})^2+5^2} \\
&\quad + \lambda_{13}\sqrt{(\alpha_1-\alpha_{13})^2+\left(\delta_{e_1}-\delta_{e13}\right)^2+(\delta_{r1}-\delta_{r13})^2+(\delta_{a1}-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$C_{L2}(\alpha_2, \delta_{e_2}, \delta_{r2}, \delta_{a2})$$

$$= \lambda_1\sqrt{(\alpha_2-\alpha_1)^2+\left(\delta_{e_2}-\delta_{e_1}\right)^2+(\delta_{r2}-\delta r_1)^2+(\delta_{a2}-\delta_{a1})^2+5^2}$$

$$+\lambda_2\sqrt{(\alpha_2-\alpha_2)^2+\left(\delta_{e_2}-\delta_{e2}\right)^2+(\delta_{r2}-\delta_{r2})^2+(\delta_{a2}-\delta_{a2})^2+5^2}$$

$$+\lambda_3\sqrt{(\alpha_2-\alpha_3)^2+\left(\delta_{e_2}-\delta_{e3}\right)^2+(\delta_{r2}-\delta_{r3})^2+(\delta_{a2}-\delta_{a3})^2+5^2}$$

$$+\lambda_4\sqrt{(\alpha_2-\alpha_4)^2+\left(\delta_{e_2}-\delta_{e4}\right)^2+(\delta_{r2}-\delta_{r4})^2+(\delta_{a2}-\delta_{a4})^2+5^2}$$

$$+\lambda_5\sqrt{(\alpha_2-\alpha_5)^2+\left(\delta_{e_2}-\delta_{e5}\right)^2+(\delta_{r2}-\delta_{r5})^2+(\delta_{a2}-\delta_{a5})^2+5^2}$$

$$+\lambda_6\sqrt{(\alpha_2-\alpha_6)^2+\left(\delta_{e_2}-\delta_{e6}\right)^2+(\delta_{r2}-\delta_{r6})^2+(\delta_{a2}-\delta_{a6})^2+5^2}$$

$$+\lambda_7\sqrt{(\alpha_2-\alpha_7)^2+\left(\delta_{e_2}-\delta_{e7}\right)^2+(\delta_{r2}-\delta_{r7})^2+(\delta_{a2}-\delta_{a7})^2+5^2}$$

$$+\lambda_8\sqrt{(\alpha_2-\alpha_8)^2+\left(\delta_{e_2}-\delta_{e8}\right)^2+(\delta_{r2}-\delta_{r8})^2+(\delta_{a2}-\delta_{a8})^2+5^2}$$

$$+\lambda_9\sqrt{(\alpha_2-\alpha_9)^2+\left(\delta_{e_2}-\delta_{e9}\right)^2+(\delta_{r2}-\delta_{r9})^2+(\delta_{a2}-\delta_{a9})^2+5^2}$$

$$+\lambda_{10}\sqrt{(\alpha_2-\alpha_{10})^2+\left(\delta_{e_2}-\delta_{e10}\right)^2+(\delta_{r2}-\delta_{r10})^2+(\delta_{a2}-\delta_{a10})^2+5^2}$$

$$+\lambda_{11}\sqrt{(\alpha_2-\alpha_{11})^2+\left(\delta_{e_2}-\delta_{e11}\right)^2+(\delta_{r2}-\delta_{r11})^2+(\delta_{a2}-\delta_{a11})^2+5^2}$$

$$+\lambda_{12}\sqrt{(\alpha_2-\alpha_{12})^2+\left(\delta_{e_2}-\delta_{e12}\right)^2+(\delta_{r2}-\delta_{r12})^2+(\delta_{a2}-\delta_{a12})^2+5^2}$$

$$+\lambda_{13}\sqrt{(\alpha_2-\alpha_{13})^2+\left(\delta_{e_2}-\delta_{e13}\right)^2+(\delta_{r2}-\delta_{r13})^2+(\delta_{a2}-\delta_{a13})^2+5^2}$$

$$C_{L3}\left(\alpha_3, \delta_{e_3}, \delta_{r3}, \delta_{a3}\right)$$

$$= \lambda_1\sqrt{(\alpha_3-\alpha_1)^2+\left(\delta_{e_3}-\delta_{e1}\right)^2+(\delta_{r3}-\delta_{r1})^2+(\delta_{a3}-\delta_{a1})^2+5^2}$$

$$+\lambda_2\sqrt{(\alpha_3-\alpha_2)^2+\left(\delta_{e_3}-\delta_{e2}\right)^2+(\delta_{r3}-\delta_{r2})^2+(\delta_{a3}-\delta_{a2})^2+5^2}$$

$$+\lambda_3\sqrt{(\alpha_3-\alpha_3)^2+\left(\delta_{e_3}-\delta_{e3}\right)^2+(\delta_{r3}-\delta_{r3})^2+(\delta_{a3}-\delta_{a3})^2+5^2}$$

$$+\lambda_4\sqrt{(\alpha_3-\alpha_4)^2+\left(\delta_{e_3}-\delta_{e4}\right)^2+(\delta_{r3}-\delta_{r4})^2+(\delta_{a3}-\delta_{a4})^2+5^2}$$

$$+\lambda_5\sqrt{(\alpha_3-\alpha_5)^2+\left(\delta_{e_3}-\delta_{e5}\right)^2+(\delta_{r3}-\delta_{r5})^2+(\delta_{a3}-\delta_{a5})^2+5^2}$$

$$+\lambda_6\sqrt{(\alpha_3-\alpha_6)^2+\left(\delta_{e_3}-\delta_{e6}\right)^2+(\delta_{r3}-\delta_{r6})^2+(\delta_{a3}-\delta_{a6})^2+5^2}$$

$$+\lambda_7\sqrt{(\alpha_3-\alpha_7)^2+\left(\delta_{e_3}-\delta_{e7}\right)^2+(\delta_{r3}-\delta_{r7})^2+(\delta_{a3}-\delta_{a7})^2+5^2}$$

$$+\lambda_8\sqrt{(\alpha_3-\alpha_8)^2+\left(\delta_{e_3}-\delta_{e8}\right)^2+(\delta_{r3}-\delta_{r8})^2+(\delta_{a3}-\delta_{a8})^2+5^2}$$

$$+\lambda_9\sqrt{(\alpha_3-\alpha_9)^2+\left(\delta_{e_3}-\delta_{e9}\right)^2+(\delta_{r3}-\delta_{r9})^2+(\delta_{a3}-\delta_{a9})^2+5^2}$$

$$+\lambda_{10}\sqrt{(\alpha_3-\alpha_{10})^2+\left(\delta_{e_3}-\delta_{e10}\right)^2+(\delta_{r3}-\delta_{r10})^2+(\delta_{a3}-\delta_{a10})^2+5^2}$$

$$+\lambda_{11}\sqrt{(\alpha_3-\alpha_{11})^2+\left(\delta_{e_3}-\delta_{e11}\right)^2+(\delta_{r3}-\delta_{r11})^2+(\delta_{a3}-\delta_{a11})^2+5^2}$$

$$+\lambda_{12}\sqrt{(\alpha_3-\alpha_{12})^2+\left(\delta_{e_3}-\delta_{e12}\right)^2+(\delta_{r3}-\delta_{r12})^2+(\delta_{a3}-\delta_{a12})^2+5^2}$$

$$+\lambda_{13}\sqrt{(\alpha_3-\alpha_{13})^2+\left(\delta_{e_3}-\delta_{e13}\right)^2+(\delta_{r3}-\delta_{r13})^2+(\delta_{a3}-\delta_{a13})^2+5^2}$$

$$
\begin{aligned}
&C_{L4}(\alpha_4, \delta_{e_4}, \delta_{r4}, \delta_{a4}) \\
&= \lambda_1\sqrt{(\alpha_4-\alpha_1)^2+\left(\delta_{e_4}-\delta_{e1}\right)^2+(\delta_{r4}-\delta_{r1})^2+(\delta_{a4}-\delta_{a1})^2+5^2} \\
&+ \lambda_2\sqrt{(\alpha_4-\alpha_2)^2+\left(\delta_{e_4}-\delta_{e2}\right)^2+(\delta_{r4}-\delta_{r2})^2+(\delta_{a4}-\delta_{a2})^2+5^2} \\
&+ \lambda_3\sqrt{(\alpha_4-\alpha_3)^2+\left(\delta_{e_4}-\delta_{e3}\right)^2+(\delta_{r4}-\delta_{r3})^2+(\delta_{a4}-\delta_{a3})^2+5^2} \\
&+ \lambda_4\sqrt{(\alpha_4-\alpha_4)^2+\left(\delta_{e_4}-\delta_{e4}\right)^2+(\delta_{r4}-\delta_{r4})^2+(\delta_{a4}-\delta_{a4})^2+5^2} \\
&+ \lambda_5\sqrt{(\alpha_4-\alpha_5)^2+\left(\delta_{e_4}-\delta_{e5}\right)^2+(\delta_{r4}-\delta_{r5})^2+(\delta_{a4}-\delta_{a5})^2+5^2} \\
&+ \lambda_6\sqrt{(\alpha_4-\alpha_6)^2+\left(\delta_{e_4}-\delta_{e6}\right)^2+(\delta_{r4}-\delta_{r6})^2+(\delta_{a4}-\delta_{a6})^2+5^2} \\
&+ \lambda_7\sqrt{(\alpha_4-\alpha_7)^2+\left(\delta_{e_4}-\delta_{e7}\right)^2+(\delta_{r4}-\delta_{r7})^2+(\delta_{a4}-\delta_{a7})^2+5^2} \\
&+ \lambda_8\sqrt{(\alpha_4-\alpha_8)^2+\left(\delta_{e_4}-\delta_{e8}\right)^2+(\delta_{r4}-\delta_{r8})^2+(\delta_{a4}-\delta_{a8})^2+5^2} \\
&+ \lambda_9\sqrt{(\alpha_4-\alpha_9)^2+\left(\delta_{e_4}-\delta_{e9}\right)^2+(\delta_{r4}-\delta_{r9})^2+(\delta_{a4}-\delta_{a9})^2+5^2} \\
&+ \lambda_{10}\sqrt{(\alpha_4-\alpha_{10})^2+\left(\delta_{e_4}-\delta_{e10}\right)^2+(\delta_{r4}-\delta_{r10})^2+(\delta_{a4}-\delta_{a10})^2+5^2} \\
&+ \lambda_{11}\sqrt{(\alpha_4-\alpha_{11})^2+\left(\delta_{e_4}-\delta_{e11}\right)^2+(\delta_{r4}-\delta_{r11})^2+(\delta_{a4}-\delta_{a11})^2+5^2} \\
&+ \lambda_{12}\sqrt{(\alpha_4-\alpha_{12})^2+\left(\delta_{e_4}-\delta_{e12}\right)^2+(\delta_{r4}-\delta_{r12})^2+(\delta_{a4}-\delta_{a12})^2+5^2} \\
&+ \lambda_{13}\sqrt{(\alpha_4-\alpha_{13})^2+\left(\delta_{e_4}-\delta_{e13}\right)^2+(\delta_{r4}-\delta_{r13})^2+(\delta_{a4}-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$C_{L5}(\alpha_5, \delta_{e_5}, \delta_{r5}, \delta_{a5})$$

$$= \lambda_1\sqrt{(\alpha_5-\alpha_1)^2+\left(\delta_{e_5}-\delta_{e1}\right)^2+(\delta_{r5}-\delta_{r1})^2+(\delta_{a5}-\delta_{a1})^2+5^2}$$

$$+\lambda_2\sqrt{(\alpha_5-\alpha_2)^2+\left(\delta_{e_5}-\delta_{e2}\right)^2+(\delta_{r5}-\delta_{r2})^2+(\delta_{a5}-\delta_{a2})^2+5^2}$$

$$+\lambda_3\sqrt{(\alpha_5-\alpha_3)^2+\left(\delta_{e_5}-\delta_{e3}\right)^2+(\delta_{r5}-\delta_{r3})^2+(\delta_{a5}-\delta_{a3})^2+5^2}$$

$$+\lambda_4\sqrt{(\alpha_5-\alpha_4)^2+\left(\delta_{e_5}-\delta_{e4}\right)^2+(\delta_{r5}-\delta_{r4})^2+(\delta_{a5}-\delta_{a4})^2+5^2}$$

$$+\lambda_5\sqrt{(\alpha_5-\alpha_5)^2+\left(\delta_{e_5}-\delta_{e5}\right)^2+(\delta_{r5}-\delta_{r5})^2+(\delta_{a5}-\delta_{a5})^2+5^2}$$

$$+\lambda_6\sqrt{(\alpha_5-\alpha_6)^2+\left(\delta_{e_5}-\delta_{e6}\right)^2+(\delta_{r5}-\delta_{r6})^2+(\delta_{a5}-\delta_{a6})^2+5^2}$$

$$+\lambda_7\sqrt{(\alpha_5-\alpha_7)^2+\left(\delta_{e_5}-\delta_{e7}\right)^2+(\delta_{r5}-\delta_{r7})^2+(\delta_{a5}-\delta_{a7})^2+5^2}$$

$$+\lambda_8\sqrt{(\alpha_5-\alpha_8)^2+\left(\delta_{e_5}-\delta_{e8}\right)^2+(\delta_{r5}-\delta_{r8})^2+(\delta_{a5}-\delta_{a8})^2+5^2}$$

$$+\lambda_9\sqrt{(\alpha_5-\alpha_9)^2+\left(\delta_{e_5}-\delta_{e9}\right)^2+(\delta_{r5}-\delta_{r9})^2+(\delta_{a5}-\delta_{a9})^2+5^2}$$

$$+\lambda_{10}\sqrt{(\alpha_5-\alpha_{10})^2+\left(\delta_{e_5}-\delta_{e10}\right)^2+(\delta_{r5}-\delta_{r10})^2+(\delta_{a5}-\delta_{a10})^2+5^2}$$

$$+\lambda_{11}\sqrt{(\alpha_5-\alpha_{11})^2+\left(\delta_{e_5}-\delta_{e11}\right)^2+(\delta_{r5}-\delta_{r11})^2+(\delta_{a5}-\delta_{a11})^2+5^2}$$

$$+\lambda_{12}\sqrt{(\alpha_5-\alpha_{12})^2+\left(\delta_{e_5}-\delta_{e12}\right)^2+(\delta_{r5}-\delta_{r12})^2+(\delta_{a5}-\delta_{a12})^2+5^2}$$

$$+\lambda_{13}\sqrt{(\alpha_5-\alpha_{13})^2+\left(\delta_{e_5}-\delta_{e13}\right)^2+(\delta_{r5}-\delta_{r13})^2+(\delta_{a5}-\delta_{a13})^2+5^2}$$

$$
\begin{aligned}
&C_{L6}(\alpha_6, \delta_{e_6}, \delta_{r6}, \delta_{a6}) \\
&= \lambda_1\sqrt{(\alpha_6-\alpha_1)^2+\left(\delta_{e_6}-\delta_{e1}\right)^2+(\delta_{r6}-\delta_{r1})^2+(\delta_{a6}-\delta_{a1})^2+5^2} \\
&+\lambda_2\sqrt{(\alpha_6-\alpha_2)^2+\left(\delta_{e_6}-\delta_{e2}\right)^2+(\delta_{r6}-\delta_{r2})^2+(\delta_{a6}-\delta_{a2})^2+5^2} \\
&+\lambda_3\sqrt{(\alpha_6-\alpha_3)^2+\left(\delta_{e_6}-\delta_{e3}\right)^2+(\delta_{r6}-\delta_{r3})^2+(\delta_{a6}-\delta_{a3})^2+5^2} \\
&+\lambda_4\sqrt{(\alpha_6-\alpha_4)^2+\left(\delta_{e_6}-\delta_{e4}\right)^2+(\delta_{r6}-\delta_{r4})^2+(\delta_{a6}-\delta_{a4})^2+5^2} \\
&+\lambda_5\sqrt{(\alpha_6-\alpha_5)^2+\left(\delta_{e_6}-\delta_{e5}\right)^2+(\delta_{r6}-\delta_{r5})^2+(\delta_{a6}-\delta_{a5})^2+5^2} \\
&+\lambda_6\sqrt{(\alpha_6-\alpha_6)^2+\left(\delta_{e_6}-\delta_{e6}\right)^2+(\delta_{r6}-\delta_{r6})^2+(\delta_{a6}-\delta_{a6})^2+5^2} \\
&+\lambda_7\sqrt{(\alpha_6-\alpha_7)^2+\left(\delta_{e_6}-\delta_{e7}\right)^2+(\delta_{r6}-\delta_{r7})^2+(\delta_{a6}-\delta_{a7})^2+5^2} \\
&+\lambda_8\sqrt{(\alpha_6-\alpha_8)^2+\left(\delta_{e_6}-\delta_{e8}\right)^2+(\delta_{r6}-\delta_{r8})^2+(\delta_{a6}-\delta_{a8})^2+5^2} \\
&+\lambda_9\sqrt{(\alpha_6-\alpha_9)^2+\left(\delta_{e_6}-\delta_{e9}\right)^2+(\delta_{r6}-\delta_{r9})^2+(\delta_{a6}-\delta_{a9})^2+5^2} \\
&+\lambda_{10}\sqrt{(\alpha_6-\alpha_{10})^2+\left(\delta_{e_6}-\delta_{e10}\right)^2+(\delta_{r6}-\delta_{r10})^2+(\delta_{a6}-\delta_{a10})^2+5^2} \\
&+\lambda_{11}\sqrt{(\alpha_6-\alpha_{11})^2+\left(\delta_{e_6}-\delta_{e11}\right)^2+(\delta_{r6}-\delta_{r11})^2+(\delta_{a6}-\delta_{a11})^2+5^2} \\
&+\lambda_{12}\sqrt{(\alpha_6-\alpha_{12})^2+\left(\delta_{e_6}-\delta_{e12}\right)^2+(\delta_{r6}-\delta_{r12})^2+(\delta_{a6}-\delta_{a12})^2+5^2} \\
&+\lambda_{13}\sqrt{(\alpha_6-\alpha_{13})^2+\left(\delta_{e_6}-\delta_{e13}\right)^2+(\delta_{r6}-\delta_{r13})^2+(\delta_{a6}-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$C_{L7}(\alpha_7, \delta_{e_7}, \delta_{r7}, \delta_{a7})$$

$$= \lambda_1\sqrt{(\alpha_7-\alpha_1)^2+\left(\delta_{e_7}-\delta_{e1}\right)^2+(\delta_{r7}-\delta_{r1})^2+(\delta_{a7}-\delta_{a1})^2+5^2}$$

$$+\lambda_2\sqrt{(\alpha_7-\alpha_2)^2+\left(\delta_{e_7}-\delta_{e2}\right)^2+(\delta_{r7}-\delta_{r2})^2+(\delta_{a7}-\delta_{a2})^2+5^2}$$

$$+\lambda_3\sqrt{(\alpha_7-\alpha_3)^2+\left(\delta_{e_7}-\delta_{e3}\right)^2+(\delta_{r7}-\delta_{r3})^2+(\delta_{a7}-\delta_{a3})^2+5^2}$$

$$+\lambda_4\sqrt{(\alpha_7-\alpha_4)^2+\left(\delta_{e_7}-\delta_{e4}\right)^2+(\delta_{r7}-\delta_{r4})^2+(\delta_{a7}-\delta_{a4})^2+5^2}$$

$$+\lambda_5\sqrt{(\alpha_7-\alpha_5)^2+\left(\delta_{e_7}-\delta_{e5}\right)^2+(\delta_{r7}-\delta_{r5})^2+(\delta_{a7}-\delta_{a5})^2+5^2}$$

$$+\lambda_6\sqrt{(\alpha_7-\alpha_6)^2+\left(\delta_{e_7}-\delta_{e6}\right)^2+(\delta_{r7}-\delta_{r6})^2+(\delta_{a7}-\delta_{a6})^2+5^2}$$

$$+\lambda_7\sqrt{(\alpha_7-\alpha_7)^2+\left(\delta_{e_7}-\delta_{e7}\right)^2+(\delta_{r7}-\delta_{r7})^2+(\delta_{a7}-\delta_{a7})^2+5^2}$$

$$+\lambda_8\sqrt{(\alpha_7-\alpha_8)^2+\left(\delta_{e_7}-\delta_{e8}\right)^2+(\delta_{r7}-\delta_{r8})^2+(\delta_{a7}-\delta_{a8})^2+5^2}$$

$$+\lambda_9\sqrt{(\alpha_7-\alpha_9)^2+\left(\delta_{e_7}-\delta_{e9}\right)^2+(\delta_{r7}-\delta_{r9})^2+(\delta_{a7}-\delta_{a9})^2+5^2}$$

$$+\lambda_{10}\sqrt{(\alpha_7-\alpha_{10})^2+\left(\delta_{e_7}-\delta_{e10}\right)^2+(\delta_{r7}-\delta_{r10})^2+(\delta_{a7}-\delta_{a10})^2+5^2}$$

$$+\lambda_{11}\sqrt{(\alpha_7-\alpha_{11})^2+\left(\delta_{e_7}-\delta_{e11}\right)^2+(\delta_{r7}-\delta_{r11})^2+(\delta_{a7}-\delta_{a11})^2+5^2}$$

$$+\lambda_{12}\sqrt{(\alpha_7-\alpha_{12})^2+\left(\delta_{e_7}-\delta_{e12}\right)^2+(\delta_{r7}-\delta_{r12})^2+(\delta_{a7}-\delta_{a12})^2+5^2}$$

$$+\lambda_{13}\sqrt{(\alpha_7-\alpha_{13})^2+\left(\delta_{e_7}-\delta_{e13}\right)^2+(\delta_{r7}-\delta_{r13})^2+(\delta_{a7}-\delta_{a13})^2+5^2}$$

$$C_{L8}\left(\alpha_8, \delta_{e_8}, \delta_{r8}, \delta_{a8}\right)$$

$$= \lambda_1\sqrt{(\alpha_8-\alpha_1)^2+\left(\delta_{e_8}-\delta_{e1}\right)^2+(\delta_{r8}-\delta_{r1})^2+(\delta_{a8}-\delta_{a1})^2+5^2}$$

$$+\lambda_2\sqrt{(\alpha_8-\alpha_2)^2+\left(\delta_{e_8}-\delta_{e2}\right)^2+(\delta_{r8}-\delta_{r2})^2+(\delta_{a8}-\delta_{a2})^2+5^2}$$

$$+\lambda_3\sqrt{(\alpha_8-\alpha_3)^2+\left(\delta_{e_8}-\delta_{e3}\right)^2+(\delta_{r8}-\delta_{r3})^2+(\delta_{a8}-\delta_{a3})^2+5^2}$$

$$+\lambda_4\sqrt{(\alpha_8-\alpha_4)^2+\left(\delta_{e_8}-\delta_{e4}\right)^2+(\delta_{r8}-\delta_{r4})^2+(\delta_{a8}-\delta_{a4})^2+5^2}$$

$$+\lambda_5\sqrt{(\alpha_8-\alpha_5)^2+\left(\delta_{e_8}-\delta_{e5}\right)^2+(\delta_{r8}-\delta_{r5})^2+(\delta_{a8}-\delta_{a5})^2+5^2}$$

$$+\lambda_6\sqrt{(\alpha_8-\alpha_6)^2+\left(\delta_{e_8}-\delta_{e6}\right)^2+(\delta_{r8}-\delta_{r6})^2+(\delta_{a8}-\delta_{a6})^2+5^2}$$

$$+\lambda_7\sqrt{(\alpha_8-\alpha_7)^2+\left(\delta_{e_8}-\delta_{e7}\right)^2+(\delta_{r8}-\delta_{r7})^2+(\delta_{a8}-\delta_{a7})^2+5^2}$$

$$+\lambda_8\sqrt{(\alpha_8-\alpha_8)^2+\left(\delta_{e_8}-\delta_{e8}\right)^2+(\delta_{r8}-\delta_{r8})^2+(\delta_{a8}-\delta_{a8})^2+5^2}$$

$$+\lambda_9\sqrt{(\alpha_8-\alpha_9)^2+\left(\delta_{e_8}-\delta_{e9}\right)^2+(\delta_{r8}-\delta_{r9})^2+(\delta_{a8}-\delta_{a9})^2+5^2}$$

$$+\lambda_{10}\sqrt{(\alpha_8-\alpha_{10})^2+\left(\delta_{e_8}-\delta_{e10}\right)^2+(\delta_{r8}-\delta_{r10})^2+(\delta_{a8}-\delta_{a10})^2+5^2}$$

$$+\lambda_{11}\sqrt{(\alpha_8-\alpha_{11})^2+\left(\delta_{e_8}-\delta_{e11}\right)^2+(\delta_{r8}-\delta_{r11})^2+(\delta_{a8}-\delta_{a11})^2+5^2}$$

$$+\lambda_{12}\sqrt{(\alpha_8-\alpha_{12})^2+\left(\delta_{e_8}-\delta_{e12}\right)^2+(\delta_{r8}-\delta_{r12})^2+(\delta_{a8}-\delta_{a12})^2+5^2}$$

$$+\lambda_{13}\sqrt{(\alpha_8-\alpha_{13})^2+\left(\delta_{e_8}-\delta_{e13}\right)^2+(\delta_{r8}-\delta_{r13})^2+(\delta_{a8}-\delta_{a13})^2+5^2}$$

$$C_{L9}(\alpha_9, \delta_{e_9}, \delta_{r9}, \delta_{a9})$$

$$= \lambda_1\sqrt{(\alpha_9-\alpha_1)^2 + \left(\delta_{e_9}-\delta_{e1}\right)^2 + (\delta_{r9}-\delta_{r1})^2 + (\delta_{a9}-\delta_{a1})^2 + 5^2}$$

$$+ \lambda_2\sqrt{(\alpha_9-\alpha_2)^2 + \left(\delta_{e_9}-\delta_{e2}\right)^2 + (\delta_{r9}-\delta_{r2})^2 + (\delta_{a9}-\delta_{a2})^2 + 5^2}$$

$$+ \lambda_3\sqrt{(\alpha_9-\alpha_3)^2 + \left(\delta_{e_9}-\delta_{e3}\right)^2 + (\delta_{r9}-\delta_{r3})^2 + (\delta_{a9}-\delta_{a3})^2 + 5^2}$$

$$+ \lambda_4\sqrt{(\alpha_9-\alpha_4)^2 + \left(\delta_{e_9}-\delta_{e4}\right)^2 + (\delta_{r9}-\delta_{r4})^2 + (\delta_{a9}-\delta_{a4})^2 + 5^2}$$

$$+ \lambda_5\sqrt{(\alpha_9-\alpha_5)^2 + \left(\delta_{e_9}-\delta_{e5}\right)^2 + (\delta_{r9}-\delta_{r5})^2 + (\delta_{a9}-\delta_{a5})^2 + 5^2}$$

$$+ \lambda_6\sqrt{(\alpha_9-\alpha_6)^2 + \left(\delta_{e_9}-\delta_{e6}\right)^2 + (\delta_{r9}-\delta_{r6})^2 + (\delta_{a9}-\delta_{a6})^2 + 5^2}$$

$$+ \lambda_7\sqrt{(\alpha_9-\alpha_7)^2 + \left(\delta_{e_9}-\delta_{e7}\right)^2 + (\delta_{r9}-\delta_{r7})^2 + (\delta_{a9}-\delta_{a7})^2 + 5^2}$$

$$+ \lambda_8\sqrt{(\alpha_9-\alpha_8)^2 + \left(\delta_{e_9}-\delta_{e8}\right)^2 + (\delta_{r9}-\delta_{r8})^2 + (\delta_{a9}-\delta_{a8})^2 + 5^2}$$

$$+ \lambda_9\sqrt{(\alpha_9-\alpha_9)^2 + \left(\delta_{e_9}-\delta_{e9}\right)^2 + (\delta_{r9}-\delta_{r9})^2 + (\delta_{a9}-\delta_{a9})^2 + 5^2}$$

$$+ \lambda_{10}\sqrt{(\alpha_9-\alpha_{10})^2 + \left(\delta_{e_9}-\delta_{e10}\right)^2 + (\delta_{r9}-\delta_{r10})^2 + (\delta_{a9}-\delta_{a10})^2 + 5^2}$$

$$+ \lambda_{11}\sqrt{(\alpha_9-\alpha_{11})^2 + \left(\delta_{e_9}-\delta_{e11}\right)^2 + (\delta_{r9}-\delta_{r11})^2 + (\delta_{a9}-\delta_{a11})^2 + 5^2}$$

$$+ \lambda_{12}\sqrt{(\alpha_9-\alpha_{12})^2 + \left(\delta_{e_9}-\delta_{e12}\right)^2 + (\delta_{r9}-\delta_{r12})^2 + (\delta_{a9}-\delta_{a12})^2 + 5^2}$$

$$+ \lambda_{13}\sqrt{(\alpha_9-\alpha_{13})^2 + \left(\delta_{e_9}-\delta_{e13}\right)^2 + (\delta_{r9}-\delta_{r13})^2 + (\delta_{a9}-\delta_{a13})^2 + 5^2}$$

$$
\begin{aligned}
&C_{L10}\left(\alpha_{10}, \delta_{e_{10}}, \delta_{r10}, \delta_{a10}\right) \\
&= \lambda_1\sqrt{(\alpha_{10}-\alpha_1)^2+\left(\delta_{e_{10}}-\delta_{e1}\right)^2+(\delta_{r10}-\delta_{r1})^2+(\delta_{a10}-\delta_{a1})^2+5^2} \\
&+\lambda_2\sqrt{(\alpha_{10}-\alpha_2)^2+\left(\delta_{e_{10}}-\delta_{e2}\right)^2+(\delta_{r10}-\delta_{r2})^2+(\delta_{a10}-\delta_{a2})^2+5^2} \\
&+\lambda_3\sqrt{(\alpha_{10}-\alpha_3)^2+\left(\delta_{e_{10}}-\delta_{e3}\right)^2+(\delta_{r10}-\delta_{r3})^2+(\delta_{a10}-\delta_{a3})^2+5^2} \\
&+\lambda_4\sqrt{(\alpha_{10}-\alpha_4)^2+\left(\delta_{e_{10}}-\delta_{e4}\right)^2+(\delta_{r10}-\delta_{r4})^2+(\delta_{a10}-\delta_{a4})^2+5^2} \\
&+\lambda_5\sqrt{(\alpha_{10}-\alpha_5)^2+\left(\delta_{e_{10}}-\delta_{e5}\right)^2+(\delta_{r10}-\delta_{r5})^2+(\delta_{a10}-\delta_{a5})^2+5^2} \\
&+\lambda_6\sqrt{(\alpha_{10}-\alpha_6)^2+\left(\delta_{e_{10}}-\delta_{e6}\right)^2+(\delta_{r10}-\delta_{r6})^2+(\delta_{a10}-\delta_{a6})^2+5^2} \\
&+\lambda_7\sqrt{(\alpha_{10}-\alpha_7)^2+\left(\delta_{e_{10}}-\delta_{e7}\right)^2+(\delta_{r10}-\delta_{r7})^2+(\delta_{a10}-\delta_{a7})^2+5^2} \\
&+\lambda_8\sqrt{(\alpha_{10}-\alpha_8)^2+\left(\delta_{e_{10}}-\delta_{e8}\right)^2+(\delta_{r10}-\delta_{r8})^2+(\delta_{a10}-\delta_{a8})^2+5^2} \\
&+\lambda_9\sqrt{(\alpha_{10}-\alpha_9)^2+\left(\delta_{e_{10}}-\delta_{e9}\right)^2+(\delta_{r10}-\delta_{r9})^2+(\delta_{a10}-\delta_{a9})^2+5^2} \\
&+\lambda_{10}\sqrt{(\alpha_{10}-\alpha_{10})^2+\left(\delta_{e_{10}}-\delta_{e10}\right)^2+(\delta_{r10}-\delta_{r10})^2+(\delta_{a10}-\delta_{a10})^2+5^2} \\
&+\lambda_{11}\sqrt{(\alpha_{10}-\alpha_{11})^2+\left(\delta_{e_{10}}-\delta_{e11}\right)^2+(\delta_{r10}-\delta_{r11})^2+(\delta_{a10}-\delta_{a11})^2+5^2} \\
&+\lambda_{12}\sqrt{(\alpha_{10}-\alpha_{12})^2+\left(\delta_{e_{10}}-\delta_{e12}\right)^2+(\delta_{r10}-\delta_{r12})^2+(\delta_{a10}-\delta_{a12})^2+5^2} \\
&+\lambda_{13}\sqrt{(\alpha_{10}-\alpha_{13})^2+\left(\delta_{e_{10}}-\delta_{e13}\right)^2+(\delta_{r10}-\delta_{r13})^2+(\delta_{a10}-\delta_{a13})^2+5^2}
\end{aligned}
$$

$$
\begin{aligned}
&C_{L11}(\alpha_{11}, \delta_{e_{11}}, \delta_{r11}, \delta_{a11}) \\
&= \lambda_1\sqrt{(\alpha_{11}-\alpha_1)^2 + \left(\delta_{e_{11}}-\delta_{e1}\right)^2 + (\delta_{r11}-\delta_{r1})^2 + (\delta_{a11}-\delta_{a1})^2 + 5^2} \\
&+ \lambda_2\sqrt{(\alpha_{11}-\alpha_2)^2 + \left(\delta_{e_{11}}-\delta_{e2}\right)^2 + (\delta_{r11}-\delta_{r2})^2 + (\delta_{a11}-\delta_{a2})^2 + 5^2} \\
&+ \lambda_3\sqrt{(\alpha_{11}-\alpha_3)^2 + \left(\delta_{e_{11}}-\delta_{e3}\right)^2 + (\delta_{r11}-\delta_{r3})^2 + (\delta_{a11}-\delta_{a3})^2 + 5^2} \\
&+ \lambda_4\sqrt{(\alpha_{11}-\alpha_4)^2 + \left(\delta_{e_{11}}-\delta_{e4}\right)^2 + (\delta_{r11}-\delta_{r4})^2 + (\delta_{a11}-\delta_{a4})^2 + 5^2} \\
&+ \lambda_5\sqrt{(\alpha_{11}-\alpha_5)^2 + \left(\delta_{e_{11}}-\delta_{e5}\right)^2 + (\delta_{r11}-\delta_{r5})^2 + (\delta_{a11}-\delta_{a5})^2 + 5^2} \\
&+ \lambda_6\sqrt{(\alpha_{11}-\alpha_6)^2 + \left(\delta_{e_{11}}-\delta_{e6}\right)^2 + (\delta_{r11}-\delta_{r6})^2 + (\delta_{a11}-\delta_{a6})^2 + 5^2} \\
&+ \lambda_7\sqrt{(\alpha_{11}-\alpha_7)^2 + \left(\delta_{e_{11}}-\delta_{e7}\right)^2 + (\delta_{r11}-\delta_{r7})^2 + (\delta_{a11}-\delta_{a7})^2 + 5^2} \\
&+ \lambda_8\sqrt{(\alpha_{11}-\alpha_8)^2 + \left(\delta_{e_{11}}-\delta_{e8}\right)^2 + (\delta_{r11}-\delta_{r8})^2 + (\delta_{a11}-\delta_{a8})^2 + 5^2} \\
&+ \lambda_9\sqrt{(\alpha_{11}-\alpha_9)^2 + \left(\delta_{e_{11}}-\delta_{e9}\right)^2 + (\delta_{r11}-\delta_{r9})^2 + (\delta_{a11}-\delta_{a9})^2 + 5^2} \\
&+ \lambda_{10}\sqrt{(\alpha_{11}-\alpha_{10})^2 + \left(\delta_{e_{11}}-\delta_{e10}\right)^2 + (\delta_{r11}-\delta_{r10})^2 + (\delta_{a11}-\delta_{a10})^2 + 5^2} \\
&+ \lambda_{11}\sqrt{(\alpha_{11}-\alpha_{11})^2 + \left(\delta_{e_{11}}-\delta_{e11}\right)^2 + (\delta_{r11}-\delta_{r11})^2 + (\delta_{a11}-\delta_{a11})^2 + 5^2} \\
&+ \lambda_{12}\sqrt{(\alpha_{11}-\alpha_{12})^2 + \left(\delta_{e_{11}}-\delta_{e12}\right)^2 + (\delta_{r11}-\delta_{r12})^2 + (\delta_{a11}-\delta_{a12})^2 + 5^2} \\
&+ \lambda_{13}\sqrt{(\alpha_{11}-\alpha_{13})^2 + \left(\delta_{e_{11}}-\delta_{e13}\right)^2 + (\delta_{r11}-\delta_{r13})^2 + (\delta_{a11}-\delta_{a13})^2 + 5^2}
\end{aligned}
$$

$$C_{L12}(\alpha_{12}, \delta_{e_{12}}, \delta_{r12}, \delta_{a12})$$

$$= \lambda_1\sqrt{(\alpha_{12}-\alpha_1)^2 + \left(\delta_{e_{12}}-\delta_{e1}\right)^2 + (\delta_{r12}-\delta_{r1})^2 + (\delta_{a12}-\delta_{a1})^2 + 5^2}$$

$$+ \lambda_2\sqrt{(\alpha_{12}-\alpha_2)^2 + \left(\delta_{e_{12}}-\delta_{e2}\right)^2 + (\delta_{r12}-\delta_{r2})^2 + (\delta_{a12}-\delta_{a2})^2 + 5^2}$$

$$+ \lambda_3\sqrt{(\alpha_{12}-\alpha_3)^2 + \left(\delta_{e_{12}}-\delta_{e3}\right)^2 + (\delta_{r12}-\delta_{r3})^2 + (\delta_{a12}-\delta_{a3})^2 + 5^2}$$

$$+ \lambda_4\sqrt{(\alpha_{12}-\alpha_4)^2 + \left(\delta_{e_{12}}-\delta_{e4}\right)^2 + (\delta_{r12}-\delta_{r4})^2 + (\delta_{a12}-\delta_{a4})^2 + 5^2}$$

$$+ \lambda_5\sqrt{(\alpha_{12}-\alpha_5)^2 + \left(\delta_{e_{12}}-\delta_{e5}\right)^2 + (\delta_{r12}-\delta_{r5})^2 + (\delta_{a12}-\delta_{a5})^2 + 5^2}$$

$$+ \lambda_6\sqrt{(\alpha_{12}-\alpha_6)^2 + \left(\delta_{e_{12}}-\delta_{e6}\right)^2 + (\delta_{r12}-\delta_{r6})^2 + (\delta_{a12}-\delta_{a6})^2 + 5^2}$$

$$+ \lambda_7\sqrt{(\alpha_{12}-\alpha_7)^2 + \left(\delta_{e_{12}}-\delta_{e7}\right)^2 + (\delta_{r12}-\delta_{r7})^2 + (\delta_{a12}-\delta_{a7})^2 + 5^2}$$

$$+ \lambda_8\sqrt{(\alpha_{12}-\alpha_8)^2 + \left(\delta_{e_{12}}-\delta_{e8}\right)^2 + (\delta_{r12}-\delta_{r8})^2 + (\delta_{a12}-\delta_{a8})^2 + 5^2}$$

$$+ \lambda_9\sqrt{(\alpha_{12}-\alpha_9)^2 + \left(\delta_{e_{12}}-\delta_{e9}\right)^2 + (\delta_{r12}-\delta_{r9})^2 + (\delta_{a12}-\delta_{a9})^2 + 5^2}$$

$$+ \lambda_{10}\sqrt{(\alpha_{12}-\alpha_{10})^2 + \left(\delta_{e_{12}}-\delta_{e10}\right)^2 + (\delta_{r12}-\delta_{r10})^2 + (\delta_{a12}-\delta_{a10})^2 + 5^2}$$

$$+ \lambda_{11}\sqrt{(\alpha_{12}-\alpha_{11})^2 + \left(\delta_{e_{12}}-\delta_{e11}\right)^2 + (\delta_{r12}-\delta_{r11})^2 + (\delta_{a12}-\delta_{a11})^2 + 5^2}$$

$$+ \lambda_{12}\sqrt{(\alpha_{12}-\alpha_{12})^2 + \left(\delta_{e_{12}}-\delta_{e12}\right)^2 + (\delta_{r12}-\delta_{r12})^2 + (\delta_{a12}-\delta_{a12})^2 + 5^2}$$

$$+ \lambda_{13}\sqrt{(\alpha_{12}-\alpha_{13})^2 + \left(\delta_{e_{12}}-\delta_{e13}\right)^2 + (\delta_{r12}-\delta_{r13})^2 + (\delta_{a12}-\delta_{a13})^2 + 5^2}$$

$$C_{L13}(\alpha_{13}, \delta_{e_{13}}, \delta_{r13}, \delta_{a13})$$

$$= \lambda_1\sqrt{(\alpha_{13}-\alpha_1)^2 + \left(\delta_{e_{13}}-\delta_{e1}\right)^2 + (\delta_{r13}-\delta_{r1})^2 + (\delta_{a13}-\delta_{a1})^2 + 5^2}$$

$$+ \lambda_2\sqrt{(\alpha_{13}-\alpha_2)^2 + \left(\delta_{e_{13}}-\delta_{e2}\right)^2 + (\delta_{r13}-\delta_{r2})^2 + (\delta_{a13}-\delta_{a2})^2 + 5^2}$$

$$+ \lambda_3\sqrt{(\alpha_{13}-\alpha_3)^2 + \left(\delta_{e_{13}}-\delta_{e3}\right)^2 + (\delta_{r13}-\delta_{r3})^2 + (\delta_{a13}-\delta_{a3})^2 + 5^2}$$

$$+ \lambda_4\sqrt{(\alpha_{13}-\alpha_4)^2 + \left(\delta_{e_{13}}-\delta_{e4}\right)^2 + (\delta_{r13}-\delta_{r4})^2 + (\delta_{a13}-\delta_{a4})^2 + 5^2}$$

$$+ \lambda_5\sqrt{(\alpha_{13}-\alpha_5)^2 + \left(\delta_{e_{13}}-\delta_{e5}\right)^2 + (\delta_{r13}-\delta_{r5})^2 + (\delta_{a13}-\delta_{a5})^2 + 5^2}$$

$$+ \lambda_6\sqrt{(\alpha_{13}-\alpha_6)^2 + \left(\delta_{e_{13}}-\delta_{e6}\right)^2 + (\delta_{r13}-\delta_{r6})^2 + (\delta_{a13}-\delta_{a6})^2 + 5^2}$$

$$+ \lambda_7\sqrt{(\alpha_{13}-\alpha_7)^2 + \left(\delta_{e_{13}}-\delta_{e7}\right)^2 + (\delta_{r13}-\delta_{r7})^2 + (\delta_{a13}-\delta_{a7})^2 + 5^2}$$

$$+ \lambda_8\sqrt{(\alpha_{13}-\alpha_8)^2 + \left(\delta_{e_{13}}-\delta_{e8}\right)^2 + (\delta_{r13}-\delta_{r8})^2 + (\delta_{a13}-\delta_{a8})^2 + 5^2}$$

$$+ \lambda_9\sqrt{(\alpha_{13}-\alpha_9)^2 + \left(\delta_{e_{13}}-\delta_{e9}\right)^2 + (\delta_{r13}-\delta_{r9})^2 + (\delta_{a13}-\delta_{a9})^2 + 5^2}$$

$$+ \lambda_{10}\sqrt{(\alpha_{13}-\alpha_{10})^2 + \left(\delta_{e_{13}}-\delta_{e10}\right)^2 + (\delta_{r13}-\delta_{r10})^2 + (\delta_{a13}-\delta_{a10})^2 + 5^2}$$

$$+ \lambda_{11}\sqrt{(\alpha_{13}-\alpha_{11})^2 + \left(\delta_{e_{13}}-\delta_{e11}\right)^2 + (\delta_{r13}-\delta_{r11})^2 + (\delta_{a13}-\delta_{a11})^2 + 5^2}$$

$$+ \lambda_{12}\sqrt{(\alpha_{13}-\alpha_{12})^2 + \left(\delta_{e_{13}}-\delta_{e12}\right)^2 + (\delta_{r13}-\delta_{r12})^2 + (\delta_{a13}-\delta_{a12})^2 + 5^2}$$

$$+ \lambda_{13}\sqrt{(\alpha_{13}-\alpha_{13})^2 + \left(\delta_{e_{13}}-\delta_{e13}\right)^2 + (\delta_{r13}-\delta_{r13})^2 + (\delta_{a13}-\delta_{a13})^2 + 5^2}$$

Similarly, we will have 13 equations each for C_d, C_{mx}, C_{my} and C_{mz}.

APPENDIX B-1

```
public class DesignOfExperiments {
static double c = 5;
public static void main(String[ ] args) {
double[ ][ ] controlVar = {
            {2, 0, 0, 0},
            {4, 0, 0, 0},
            {6, 0, 0, 0},
            {0, -10, 0, 0},
            {0, -5, 0, 0},
            {0, 5, 0, 0},
            {0, 0, -10, 0},
            {0, 0, 5, 0},
            {0, 0, 10, 0},
            {0, 0, 0, -10},
            {0, 0, 0, -5},
            {0, 0, 0, 5},
            {0, 0, 0, 10}
      };
double[ ] unControlVar = {
            0.061447, 0.13503, 0.20972, -0.0051742, 0.020329,
0.0002129, -0.014452, -0.013721, -0.013974, 0.013095, -
0.013329, -0.012402, -0.012574
    };


int N = 13;
double[ ][ ] A = new double[N][N];
double[ ] B = new double[N];
for (int i = 0; i < N; i++) {
```

```
for (int j = 0; j < N; j++) {
double[ ] var1 = new double[controlVar[0].length];
double[ ] var2 = new double[controlVar[0].length];
for (int k = 0; k < var1.length; k++) {
var1[k] = controlVar[i][k];
var2[k] = controlVar[j][k];
                }
                A[i][j] = rbf(var1, var2);
            }
            B[i] = unControlVar[i];
        }

double[ ] lambda = gaussElim(A, B);
for (int i = 0; i < N; i++) {
System.out.println(lambda[i]);
        }
doubleval = 0.0;
double[ ] var = {2, 0, 0, 0};
for (int i = 0; i < N; i++) {
val += lambda[i] * rbf(var, controlVar[i]);
        }
System.out.println(val);
    }
public static double rbf(double[ ] v1, double[] v2) {
doublerbfValue = 0.0;
for (int i = 0; i < v1.length; i++) {
rbfValue += Math.pow(v1[i] - v2[i], 2.0);
        }
rbfValue = Math.sqrt(rbfValue + c * c);
returnrbfValue;
```

```
    }

    /**
     * Matrix solver for a system of linear equations given in matrix form
     * <code>A.X = B</code><br> Warning: Matrix A and B are changed inside this
     * method, therefore make a copy before passing to this function if A & B
     * are to be used in code further.
     *
     * @param A Square matrix
     * @param B 1-D array
     * @return an 1-D array containing solution, i.e. calculates and returns 1-D
     * vector X in the system
     * <code>A.X = B</code>
     */
private static double[] gaussElim(double A[][], double B[]) {
final double EPS = 0.000001;
double a, sum;
int size = B.length;
double[] X = new double[size];
int i, j, k;
intpivotRow, row, col;
doublemaxValue, tempValue;

        // Elimination with pivoting
        // Can be used even for non-dominat diagonal matrix
for (i = 0; i < size; i++) {
            // Pivoting procedure starts here
pivotRow = i;
```

```
maxValue = Math.abs(A[i][i]);
for (row = i + 1; row < size; row++) {
if (maxValue<Math.abs(A[row][i])) {
maxValue = Math.abs(A[row][i]);
pivotRow = row;
                    }
                }
if (pivotRow != i) {
for (col = i; col < size; col++) {
tempValue = A[i][col];
                        A[i][col] = A[pivotRow][col];
                        A[pivotRow][col] = tempValue;
                    }
tempValue = B[i];
                    B[i] = B[pivotRow];
                    B[pivotRow] = tempValue;
                } // Pivoting procedure ends here

for (j = i + 1; j < size; j++) {
if (Math.abs(A[i][i]) < EPS) { // Check sigularity
System.out.println("The matrix is singular..");
throw new RuntimeException("The matrix is singular");
                    }
                    a = A[j][i] / A[i][i];
                    A[j][i] = 0.0;
for (k = i + 1; k < size; k++) {
                        A[j][k] = A[j][k] - A[i][k] * a;
                    }
                    B[j] = B[j] - B[i] * a;
                }
```

```
            }

            // Back substitution
        for (i = size - 1; i >= 0; i--) {
        sum = B[i];
        for (j = i + 1; j < size; j++) {
        sum -= A[i][j] * X[j];
                }
                X[i] = sum / A[i][i];
            }

            // Setting matrix B as the solution
            // for (i = 0; i < size; i++)
            // B[i] = X[i];

        return X;
            }
        }
```

APPENDIX B-2

```
print("alpha\tde\tda\tdr")


for alpha in range(2, 7):

for de in range(-10, 11, 2):

for da in range(-10, 11, 2):

fordr in range(-10, 11, 2):

print("" + str(alpha) + "\t" + str(de) + "\t" + str(da) + "\t"
+ str(dr))
```

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